Exponential and Radical Functions

Population Explosion

The concepts in this chapter are used to model many real-world phenomena, such as changes in wildlife populations.

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Chapter Project Online

KEYWORD: MA7 ChProj

ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

1. like terms
2. square root
3. domain
4. perfect square
5. exponent

A. the set of second elements of a relation

B. terms that contain the same variable raised to the same power

C. the set of first elements of a relation

D. a number that tells how many times a base is used as a factor

E. a number whose positive square root is a whole number

F. one of two equal factors of a number

✓ Evaluate Powers

Find the value of each expression.

6. 2^4
7. 5^0
8. $7 \cdot 3^2$
9. $3 \cdot 5^3$
10. 3^5
11. $-6^2 + 8^1$
12. $40 \cdot 2^3$
13. $7^2 \cdot 3^1$

✓ Graph Functions

Graph each function.

14. $y = 8$
15. $y = x + 3$
16. $y = x^2 - 4$
17. $y = x^2 + 2$

✓ Fractions, Decimals, and Percents

Write each percent as a decimal.

18. 50%
19. 25%
20. 15.2%
21. 200%
22. 1.9%
23. 0.3%
24. 0.1%
25. 1.04%

✓ Squares and Square Roots

Find each square root.

26. $\sqrt{36}$
27. $\sqrt{81}$
28. $\sqrt{25}$
29. $\sqrt{64}$

✓ Pythagorean Theorem

Find the length of the hypotenuse in each right triangle.

30.

3 cm

4 cm

31.

5 in.

12 in.

32.

6 ft   8 ft
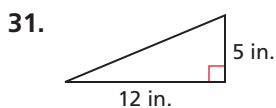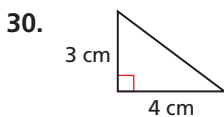
✓ Multiply Monomials and Polynomials

Multiply.

33. $5(2m - 3)$
34. $3x(8x + 9)$
35. $2t(3t - 1)$
36. $4r(4r - 5)$

Study Guide: Preview

Where You've Been

Previously, you

- identified and extended arithmetic sequences.
- identified and graphed linear functions and quadratic functions.
- solved linear and quadratic equations.

In This Chapter

You will study

- another type of sequence— geometric sequences.
- two more types of functions— exponential functions and square-root functions.
- radical equations.

Where You're Going

You can use the skills in this chapter

- to analyze more complicated functions in later math courses, such as Calculus.
- to explore exponential growth and decay models that are used in science.
- to make informed decisions about finances.

Key Vocabulary/Vocabulario

common ratio	razón común
compound interest	interés compuesto
exponential decay	decrecimiento exponencial
exponential function	función exponencial
exponential growth	crecimiento exponencial
extraneous solution	solución extraña
geometric sequence	sucesión geométrica
like radicals	radicales semejantes
radical equation	ecuación radical
radical expression	expresión radical
radicand	radicando
square-root function	función de raíz cuadrada

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What does it mean when several items have something "in common"? What is a ratio? What do you think **common ratio** means?

2. In the division problem $2\overline{)50}$, with quotient 25, the *dividend* is 50. If a *radicand* is similar to a dividend, then what is the **radicand** in $\sqrt{16} = 4$?

3. A square-root sign is also known as a *radical*. Use this knowledge to define **radical expression** and **radical equation**.

4. The root word of *extraneous* is *extra*. *Extraneous* means *irrelevant* or *unrelated*. Use this information to define **extraneous solution**.

Reading and Writing Math

Study Strategy: Remember Formulas

In math, there are many formulas, properties, and rules that you should commit to memory.

To memorize a formula, create flash cards. Write the name of the formula on one side of a card. Write the formula on the other side of the card. You might also include a diagram or an example if helpful. Study your flash cards on a regular basis.

Sample Flash Card

Front

ODDS
IN FAVOR
OF AN EVENT

Back

odds in favor =

$\dfrac{\text{number of ways an event can happen}}{\text{number of ways an event can } \underline{\text{fail}} \text{ to happen}}$

odds in favor = $a:b$
a represents the number of ways an event can happen.
b represents the number of ways an event can <u>fail</u> to happen.

Knowing when and how to apply a mathematical formula is as important as memorizing the formula itself.

To know what formula to apply, read the problem carefully and look for key words.

From Lesson 10-6

> The **probability** of choosing an ace from a deck of cards is $\frac{1}{13}$.
> What are the **odds of choosing an ace**?

The key words have been highlighted. The probability is given, and you are asked to find the odds. You should use the formula for *odds in favor of an event*.

Try This

Read each problem. Then write the formula(s) needed to solve it. What key words helped you identify the formula?

1. A manufacturer inspects 450 computer chips and finds that 22 are defective. What is the experimental probability that a chip chosen at random is defective?

2. The area of a rectangular pool is 120 square feet. The length is 1 foot less than twice the width. What is the perimeter of the pool?

11-1 Geometric Sequences

Who uses this?
Bungee jumpers can use geometric sequences to calculate how high they will bounce.

The table shows the heights of a bungee jumper's bounces.

Bounce	1	2	3
Height (ft)	200	80	32

The height of the bounces shown in the table above form a *geometric sequence*. In a **geometric sequence**, the ratio of successive terms is the same number r, called the **common ratio**.

EXAMPLE 1 Extending Geometric Sequences

Find the next three terms in each geometric sequence.

A 1, 3, 9, 27, …

Step 1 Find the value of r by dividing each term by the one before it.

1   3   9   27

$\frac{3}{1} = 3$ $\frac{9}{3} = 3$ $\frac{27}{9} = 3$ ← The value of r is 3.

Step 2 Multiply each term by 3 to find the next three terms.

27   81   243   729

× 3   × 3   × 3

The next three terms are 81, 243, and 729.

B $-16, 4, -1, \frac{1}{4}, …$

Step 1 Find the value of r by dividing each term by the one before it.

-16 4 -1 $\frac{1}{4}$

$\frac{4}{-16} = -\frac{1}{4}$ $\frac{-1}{4} = -\frac{1}{4}$ $\frac{\frac{1}{4}}{-1} = -\frac{1}{4}$ ← The value of r is $-\frac{1}{4}$.

Step 2 Multiply each term by $-\frac{1}{4}$ to find the next three terms.

$\frac{1}{4}$ $-\frac{1}{16}$ $\frac{1}{64}$ $-\frac{1}{256}$

$\times \left(-\frac{1}{4}\right)$ $\times \left(-\frac{1}{4}\right)$ $\times \left(-\frac{1}{4}\right)$

The next three terms are $-\frac{1}{16}, \frac{1}{64}$, and $-\frac{1}{256}$.

Helpful Hint

When the terms in a geometric sequence alternate between positive and negative, the value of r is negative.

Find the next three terms in each geometric sequence.

1a. $5, -10, 20, -40, \ldots$ **1b.** $512, 384, 288, \ldots$

Writing Math

The variable a is often used to represent terms in a sequence. The variable a_4 (read "a sub 4") is the fourth term in a sequence.

Geometric sequences can be thought of as functions. The term number, or position in the sequence, is the input of the function, and the term itself is the output of the function.

1	2	3	4	$\leftarrow$ **Position**
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
3	6	12	24	$\leftarrow$ **Term**
a_1	a_2	a_3	a_4	

To find the output a_n of a geometric sequence when n is a large number, you need an equation, or function rule. Look for a pattern to find a function rule for the sequence above.

Words	Numbers	Algebra
1st term	3	a_1
2nd term	$3 \cdot 2^1 = 6$	$a_1 \cdot r^1$
3rd term	$3 \cdot 2^2 = 12$	$a_1 \cdot r^2$
4th term	$3 \cdot 2^3 = 24$	$a_1 \cdot r^3$
nth term	$3 \cdot 2^{n-1}$	$a_1 \cdot r^{n-1}$

The pattern in the table shows that to get the nth term, multiply the first term by the common ratio raised to the power $n - 1$.

If the first term of a geometric sequence is a_1, the nth term is a_n, and the common ratio is r, then

$$a_n = a_1 r^{n-1}$$

nth term   1st term   Common ratio

E X A M P L E 2 **Finding the nth Term of a Geometric Sequence**

A The first term of a geometric sequence is 128, and the common ratio is 0.5. What is the 10th term of the sequence?

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_{10} = 128(0.5)^{10-1}$ *Substitute 128 for a_1, 10 for n, and 0.5 for r.*

$= 128(0.5)^9$ *Simplify the exponent.*

$= 0.25$ *Use a calculator.*

The 10th term of the sequence is 0.25.

B For a geometric sequence, $a_1 = 8$ and $r = 3$. Find the 5th term of this sequence.

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_5 = 8(3)^{5-1}$ *Substitute 8 for a_1, 5 for n, and 3 for r.*

$= 8(3)^4$ *Simplify the exponent.*

$= 648$ *Use a calculator.*

The 5th term of the sequence is 648.

C What is the 13th term of the geometric sequence 8, -16, 32, -64, ... ?

$$8 \qquad -16 \qquad 32 \qquad -64$$

$$\frac{-16}{8} = -2 \qquad \frac{32}{-16} = -2 \qquad \frac{-64}{32} = -2 \qquad \textit{The value of r is } -2.$$

$$a_n = a_1 r^{n-1} \qquad \textit{Write the formula.}$$

$$a_{13} = 8(-2)^{13-1} \qquad \textit{Substitute 8 for } a_1, \text{ 13 for n, and } -2 \text{ for r.}$$

$$= 8(-2)^{12} \qquad \textit{Simplify the exponent.}$$

$$= 32{,}768 \qquad \textit{Use a calculator.}$$

The 13th term of the sequence is 32,768.

CHECK IT OUT! **2.** What is the 8th term of the sequence 1000, 500, 250, 125, ... ?

EXAMPLE 3 *Sports Application*

A bungee jumper jumps from a bridge. The diagram shows the bungee jumper's height above the ground at the top of each bounce. The heights form a geometric sequence. What is the bungee jumper's height at the top of the 5th bounce?

First bounce 200 ft

Second bounce 80 ft

Third bounce 32 ft

$$200 \qquad 80 \qquad 32$$

$$\frac{80}{200} = 0.4 \qquad \frac{32}{80} = 0.4$$

$$a_n = a_1 r^{n-1} \qquad \textit{Write the formula.}$$

$$a_5 = 200(0.4)^{5-1} \qquad \textit{Substitute 200 for } a_1, \text{ 5 for n, and 0.4 for r.}$$

$$= 200(0.4)^4 \qquad \textit{Simplify the exponent.}$$

$$= 5.12 \qquad \textit{Use a calculator.}$$

The height of the 5th bounce is 5.12 feet.
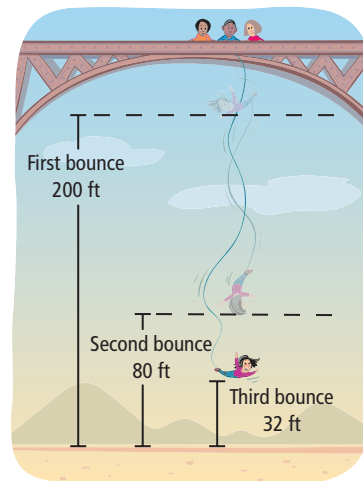
CHECK IT OUT! **3.** The table shows a car's value for 3 years after it is purchased. The values form a geometric sequence. How much will the car be worth in the 10th year?

Year	Value ($)
1	10,000
2	8,000
3	6,400

THINK AND DISCUSS

1. How do you determine whether a sequence is geometric?

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a way to represent the geometric sequence.

Ways to Represent Geometric Sequence 1, 2, 4, 8, ...

Table   Formula   Words

11-1 **Exercises**

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Homework Help Online
KEYWORD: MA7 11-1
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** What is the *common ratio* of a geometric sequence?

SEE EXAMPLE **1**
p. 766

Find the next three terms in each geometric sequence.

2. 2, 4, 8, 16, …

3. 400, 200, 100, 50, …

4. 4, −12, 36, −108, …

SEE EXAMPLE **2**
p. 767

5. The first term of a geometric sequence is 1, and the common ratio is 10. What is the 10th term of the sequence?

6. What is the 11th term of the geometric sequence 3, 6, 12, 24, … ?

SEE EXAMPLE **3**
p. 768

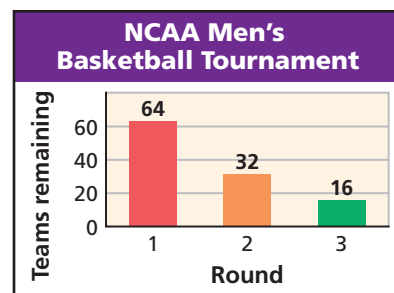7. **Sports** In the NCAA men's basketball tournament, 64 teams compete in round 1. Fewer teams remain in each following round, as shown in the graph, until all but one team have been eliminated. The numbers of teams in each round form a geometric sequence. How many teams compete in round 5?

NCAA Men's Basketball Tournament

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–13	1
14–15	2
16	3

Extra Practice
Skills Practice p. S24
Application Practice p. S38

Find the next three terms in each geometric sequence.

8. −2, 10, −50, 250, …

9. 32, 48, 72, 108, …

10. 625, 500, 400, 320, …

11. 6, 42, 294, …

12. 6, −12, 24, −48, …

13. $40, 10, \frac{5}{2}, \frac{5}{8}, \ldots$

14. The first term of a geometric sequence is 18 and the common ratio is 3.5. What is the 5th term of the sequence?

15. What is the 14th term of the geometric sequence 1000, 100, 10, 1, … ?

16. **Physical Science** A ball is dropped from a height of 500 meters. The table shows the height of each bounce, and the heights form a geometric sequence. How high does the ball bounce on the 8th bounce? Round your answer to the nearest tenth of a meter.

Bounce	Height (m)
1	400
2	320
3	256

Find the missing term(s) in each geometric sequence.

17. 20, 40, ▮, ▮, …

18. ▮, 6, 18, ▮, …

19. 9, 3, 1, ▮, …

20. 3, 12, ▮, 192, ▮, …

21. $7, 1, ▮, ▮, \frac{1}{343}, \ldots$

22. $▮, 100, 25, ▮, \frac{25}{16}, \ldots$

23. −3, ▮, −12, 24, ▮, …

24. ▮, ▮, 1, −3, 9, …

25. 1, 17, 289, ▮, …

Determine whether each sequence could be geometric. If so, give the common ratio.

26. 2, 10, 50, 250, …

27. $15, 5, \frac{5}{3}, \frac{5}{9}, \ldots$

28. 6, 18, 24, 38, …

29. 9, 3, −1, −5, …

30. 7, 21, 63, 189, …

31. 4, 1, −2, −4, …

32. Multi-Step Billy earns money by mowing lawns for the summer. He offers two payment plans, as shown at right.

  a. Do the payments for plan 2 form a geometric sequence? Explain.

  b. If you were one of Billy's customers, which plan would you choose? (Assume that the summer is 10 weeks long.) Explain your choice.

33. Measurement When you fold a piece of paper in half, the thickness of the folded piece is twice the thickness of the original piece. A piece of copy paper is about 0.1 mm thick.

  a. How thick is a piece of copy paper that has been folded in half 7 times?

  b. Suppose that you could fold a piece of copy paper in half 12 times. How thick would it be? Write your answer in centimeters.

Geometry

Fractals have been used to model stock market behavior, the distribution of galaxies throughout the universe, and problems related to fluid turbulence. They have also been used in computer graphics to create realistic images of irregular objects.

List the first four terms of each geometric sequence.

34. $a_1 = 3, a_n = 3(2)^{n-1}$ **35.** $a_1 = -2, a_n = -2(4)^{n-1}$ **36.** $a_1 = 5, a_n = 5(-2)^{n-1}$

37. $a_1 = 2, a_n = 2(2)^{n-1}$ **38.** $a_1 = 2, a_n = 2(5)^{n-1}$ **39.** $a_1 = 12, a_n = 12\left(\frac{1}{4}\right)^{n-1}$

40. Critical Thinking What happens to the terms of a geometric sequence when r is doubled? Use an example to support your answer.

41. Geometry Fractals are geometric figures that are formed by repeating the same process over and over on a smaller and smaller scale. These are the steps to draw a square fractal.

  Step 1 (stage 0) Draw a large square.

  Step 2 (stage 1) Divide the square into four equal squares.

  Step 3 (stage 2) Divide each small square into four equal squares.

  Step 4 Repeat Step 3 indefinitely.

  a. Draw stages 0, 1, 2, and 3 of the square fractal.

  b. How many small squares are in each stage? Organize your data relating stage and number of small squares in a table.

  c. Does the data in part **b** form a geometric sequence? Explain.

  d. Write a rule to find the number of small squares in stage n.

42. Write About It Write a series of steps for finding the nth term of a geometric sequence when you are given the first several terms.

MULTI-STEP TEST PREP

43. This problem will prepare you for the Multi-Step Test Prep on page 796.

  a. Three years ago, the annual tuition at a university was $3000. The following year, the tuition was $3300, and last year, the tuition was $3630. If the tuition has continued to grow in the same manner, what is the tuition this year? What do you expect it to be next year?

  b. What is the common ratio?

 **c.** What would you predict the tuition was 4 years ago? How did you find that value?

44. Which of the following is a geometric sequence?

Ⓐ $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, ...   Ⓒ 3, 8, 13, 18, ...

Ⓑ −2, −6, −10, −14, ...   Ⓓ 5, 10, 20, 40, ...

45. Which equation represents the nth term in the geometric sequence 2, −8, 32, −128, ...?

Ⓕ $a_n = (-4)^n$ Ⓖ $a_n = (-4)^{n-1}$ Ⓗ $a_n = 2(-4)^n$ Ⓙ $a_n = 2(-4)^{n-1}$

46. The frequency of a musical note, measured in hertz (Hz), is called its pitch. The pitches of the A keys on a piano form a geometric sequence, as shown.

A_1	A_2	A_3	A_4
55 Hz	110 Hz	220 Hz	440 Hz
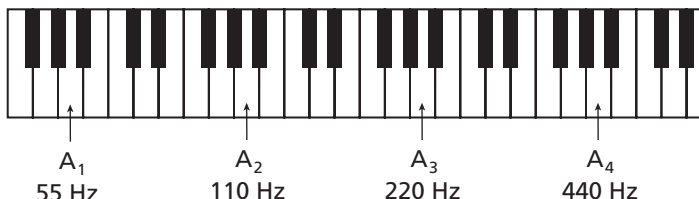
What is the frequency of A_7?

Ⓐ 880 Hz   Ⓑ 1760 Hz   Ⓒ 3520 Hz   Ⓓ 7040 Hz

CHALLENGE AND EXTEND

Find the next three terms in each geometric sequence.

47. x, x^2, x^3, ...

48. $2x^2$, $6x^3$, $18x^4$, ...

49. $\frac{1}{y^3}$, $\frac{1}{y^2}$, $\frac{1}{y}$, ...

50. $\frac{1}{(x+1)^2}$, $\frac{1}{x+1}$, 1, ...

51. The 10th term of a geometric sequence is 0.78125. The common ratio is −0.5. Find the first term of the sequence.

52. The first term of a geometric sequence is 12 and the common ratio is $\frac{1}{2}$. Is 0 a term in this sequence? Explain.

53. A geometric sequence starts with 14 and has a common ration of 0.4. Colin finds that another number in the sequence is 0.057344. Which term in the sequence did Colin find?

54. The first three terms of a sequence are 1, 2, and 4. Susanna said the 8th term of this sequence is 128. Paul said the 8th term is 29. Explain how the students found their answers. Why could these both be considered correct answers?

SPIRAL REVIEW

Solve each inequality and graph the solutions. *(Lesson 3-2)*

55. $b - 4 > 6$

56. $-12 + x \le -8$

57. $c + \frac{2}{3} < \frac{1}{3}$

Graph the solutions of each linear inequality. *(Lesson 6-5)*

58. $y < 2x - 4$

59. $3x + y > 6$

60. $-y \le 2x + 1$

Write a function to describe each of the following graphs. *(Lesson 9-4)*

61. The graph of $f(x) = x^2 - 3$ translated 7 units up

62. The graph of $f(x) = 2x^2 + 6$ narrowed and translated 2 units down

11-2 Exponential Functions

Objectives
Evaluate exponential functions.

Identify and graph exponential functions.
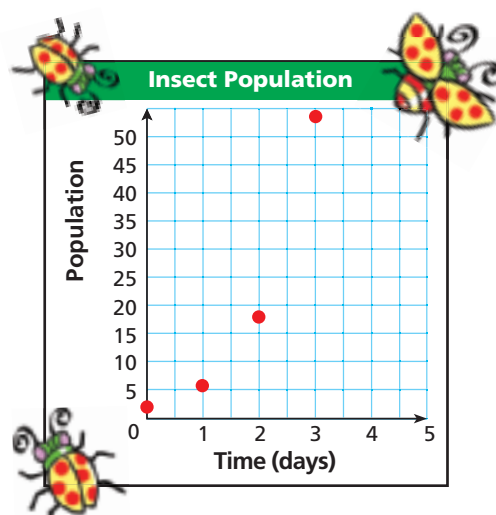
Vocabulary
exponential function

Who uses this?
Scientists model populations with exponential functions.

The table and the graph show an insect population that increases over time.

Time (days)	Population
0	**2**
1	6
2	18
3	54

× 3
× 3
× 3

Insect Population

A function rule that describes the pattern above is $f(x) = 2(3)^x$. This type of function, in which the independent variable appears in an exponent, is an **exponential function**. Notice that **2** is the starting population and **3** is the amount by which the population is multiplied each day.

> **Know it! Note**
>
> **Exponential Functions**
>
> An exponential function has the form $f(x) = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$.

EXAMPLE 1 **Evaluating an Exponential Function**

A The function $f(x) = 2(3)^x$ models an insect population after x days. What will the population be on the 5th day?

$f(x) = 2(3)^x$ *Write the function.*
$f(5) = 2(3)^5$ *Substitute 5 for x.*
$\quad\quad = 2(243)$ *Evaluate 3^5.*
$\quad\quad = 486$ *Multiply.*

There will be 486 insects on the 5th day.

Helpful Hint

In Example 1B, round your answer to the nearest whole number because there can only be a whole number of prairie dogs.

B The function $f(x) = 1500(0.995)^x$, where x is the time in years, models a prairie dog population. How many prairie dogs will there be in 8 years?

$f(x) = 1500(0.995)^x$
$f(8) = 1500(0.995)^8$ *Substitute 8 for x.*
$\quad\quad \approx 1441$ *Use a calculator. Round to the nearest whole number.*

There will be about 1441 prairie dogs in 8 years.

CHECK IT OUT!

1. The function $f(x) = 8(0.75)^x$ models the width of a photograph in inches after it has been reduced by 25% x times. What is the width of the photograph after it has been reduced 3 times?

Remember that linear functions have constant first differences and quadratic functions have constant second differences. Exponential functions do not have constant differences, but they do have *constant ratios*.

As the *x*-values increase by a constant amount, the *y*-values are multiplied by a constant amount. This amount is the constant ratio and is the value of *b* in $f(x) = ab^x$.

x	$f(x) = 2(3)^x$
1	6
2	18
3	54
4	162

+1, +1, +1 (left); ×3, ×3, ×3 (right)

EXAMPLE 2 **Identifying an Exponential Function**

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

A $\{(-1, 1.5), (0, 3), (1, 6), (2, 12)\}$

x	y
−1	1.5
0	3
1	6
2	12

+1, +1, +1 (left); ×2, ×2, ×2 (right)

This is an exponential function. As the *x*-values increase by a constant amount, the *y*-values are multiplied by a constant amount.

B $\{(-1, -9), (1, 9), (3, 27), (5, 45)\}$

x	y
−1	−9
1	9
3	27
5	45

+2, +2, +2 (left); ×(−1), ×3, ×$\frac{5}{3}$ (right)

This is *not* an exponential function. As the *x*-values increase by a constant amount, the *y*-values are *not* multiplied by a constant amount.

CHECK IT OUT! Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

2a. $\{(-1, 1), (0, 0), (1, 1), (2, 4)\}$ **2b.** $\{(-2, 4), (-1, 2), (0, 1), (1, 0.5)\}$

To graph an exponential function, choose several values of *x* (positive, negative, and 0) and generate ordered pairs. Plot the points and connect them with a smooth curve.
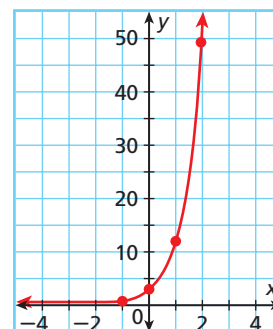
EXAMPLE 3 **Graphing $y = ab^x$ with $a > 0$ and $b > 1$**

Graph $y = 3(4)^x$.

Choose several values of x and generate ordered pairs.

x	$y = 3(4)^x$
−1	0.75
0	3
1	12
2	48

Graph the ordered pairs and connect with a smooth curve.

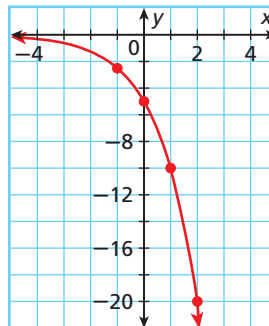**CHECK IT OUT!** **3a.** Graph $y = 2^x$. **3b.** Graph $y = 0.2(5)^x$.

EXAMPLE 4 **Graphing $y = ab^x$ with $a < 0$ and $b > 1$**

Graph $y = -5(2)^x$.

Choose several values of x and generate ordered pairs.

x	$y = -5(2)^x$
−1	−2.5
0	−5
1	−10
2	−20

Graph the ordered pairs and connect with a smooth curve.

CHECK IT OUT! 4a. Graph $y = -6^x$. 4b. Graph $y = -3(3)^x$.

EXAMPLE 5 **Graphing $y = ab^x$ with $0 < b < 1$**

Graph each exponential function.

A $y = 3\left(\dfrac{1}{2}\right)^x$

Choose several values of x and generate ordered pairs.

x	$y = 3\left(\dfrac{1}{2}\right)^x$
−1	6
0	3
1	1.5
2	0.75

Graph the ordered pairs and connect with a smooth curve.
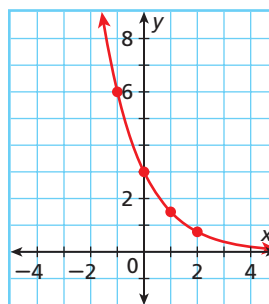
B $y = -2(0.4)^x$

Choose several values of x and generate ordered pairs.

x	$y = -2(0.4)^x$
−2	−12.5
−1	−5
0	−2
1	−0.8

Graph the ordered pairs and connect with a smooth curve.
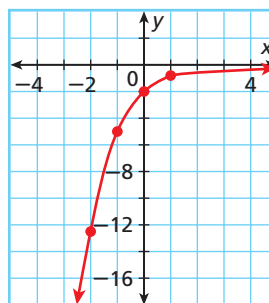
CHECK IT OUT! Graph each exponential function.

5a. $y = 4\left(\dfrac{1}{4}\right)^x$ 5b. $y = -2(0.1)^x$

The box summarizes the general shapes of exponential function graphs.

Know it! Note

Graphs of Exponential Functions	
For $y = ab^x$, if $b > 1$, then the graph will have one of these shapes.	For $y = ab^x$, if $0 < b < 1$, then the graph will have one of these shapes.

EXAMPLE 6 *Statistics Application*

In the year 2000, the world population was about 6 billion, and it was growing by 1.21% each year. At this growth rate, the function $f(x) = 6(1.0121)^x$ gives the 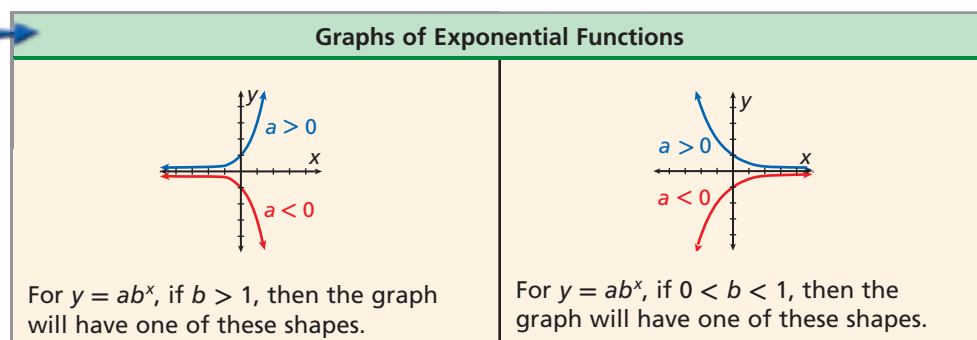population, in billions, x years after 2000. Using this model, in about what year will the population reach 7 billion?

*Enter the function into the **Y=** editor of a graphing calculator.*

Caution!

The function values give the population *in billions*, so a *y*-value of 7 means 7 billion.
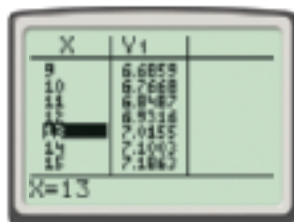
TABLE
Press **2nd** **GRAPH** . *Use the arrow keys to find a y-value as close to 7 as possible. The corresponding x-value is 13.*

The world population will reach 7 billion in about 2013.

CHECK IT OUT!

6. An accountant uses $f(x) = 12{,}330(0.869)^x$, where x is the time in years since the purchase, to model the value of a car. When will the car be worth $2000?

THINK AND DISCUSS

1. How can you find the constant ratio of a set of exponential data?

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example of an appropriate exponential function and sketch its graph.

Exponential Functions: $y = ab^x$

| $a > 0, b > 1$ | $a < 0, b > 1$ | $a > 0, 0 < b < 1$ | $a < 0, 0 < b < 1$ |

11-2 **Exercises**

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GUIDED PRACTICE

1. **Vocabulary** Tell whether $y = 3x^4$ is an *exponential function*. Explain your answer.

SEE EXAMPLE 1
p. 772

2. **Physics** The function $f(x) = 50{,}000(0.975)^x$, where x represents the underwater depth in meters, models the intensity of light below the water's surface in lumens per square meter. What is the intensity of light 200 meters below the surface? Round your answer to the nearest whole number.

SEE EXAMPLE 2
p. 773

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

3. $\{(-1, -1), (0, 0), (1, -1), (2, -4)\}$ 4. $\{(0, 1), (1, 4), (2, 16), (3, 64)\}$

SEE EXAMPLE 3
p. 773

Graph each exponential function.

5. $y = 3^x$ 6. $y = 5^x$

7. $y = 10(3)^x$ 8. $y = 5(2)^x$

SEE EXAMPLE 4
p. 774

9. $y = -2(3)^x$ 10. $y = -4(2)^x$

11. $y = -3(2)^x$ 12. $y = -2(3)^x$

SEE EXAMPLE 5
p. 774

13. $y = -\left(\frac{1}{4}\right)^x$ 14. $y = \left(\frac{1}{3}\right)^x$

15. $y = 2\left(\frac{1}{4}\right)^x$ 16. $y = -2(0.25)^x$

SEE EXAMPLE 6
p. 775

17. The function $f(x) = 57.8(1.02)^x$ gives the number of passenger cars, in millions, in the United States x years after 1960. Using this model, in about what year will the number of passenger cars reach 200 million?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–20	1
21–24	2
25–27	3
28–30	4
31–33	5
34	6

Extra Practice
Skills Practice p. S24
Application Practice p. S38

18. **Sports** If a golf ball is dropped from a height of 27 feet, the function $f(x) = 27\left(\frac{2}{3}\right)^x$ gives the height in feet of each bounce, where x is the bounce number. What will be the height of the 4th bounce?

19. Suppose the depth of a lake can be described by the function $y = 334(0.976)^x$, where x represents the number of weeks from today. Today, the depth of the lake is 334 ft. What will the depth be in 6 weeks? Round your answer to the nearest whole number.

20. **Physics** A ball rolling down a slope travels continuously faster. Suppose the function $y = 1.3(1.41)^x$ describes the speed of the ball in inches per minute. How fast will the ball be rolling in 15 minutes? Round your answer to the nearest hundredth.

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

21. $\left\{(-2, 9), (-1, 3), (0, 1), \left(1, \frac{1}{3}\right)\right\}$ 22. $\{(-1, 0), (0, 1), (1, 4), (2, 9)\}$

23. $\{(-1, -5), (0, -3), (1, -1), (2, 1)\}$ 24. $\{(-3, 6.25), (-2, 12.5), (-1, 25), (0, 50)\}$

Graph each exponential function.

25. $y = 1.5^x$

26. $y = \frac{1}{3}(3)^x$

27. $y = 100(0.7)^x$

28. $y = -2(4)^x$

29. $y = -1(5)^x$

30. $y = -\frac{1}{2}(4)^x$

31. $y = 4\left(\frac{1}{2}\right)^x$

32. $y = -2\left(\frac{1}{3}\right)^x$

33. $y = 0.5(0.25)^x$

34. Technology Moore's law states that the maximum number of transistors that can fit on a silicon chip doubles every two years. The function $f(x) = 42(1.41)^x$ models the number of transistors, in millions, that can fit on a chip, where x is the number of years since 2000. Predict what year it will be when a chip can hold 1 billion transistors.

35. Multi-Step A computer randomly creates three different functions. The functions are $y = (3.1x + 7)^2$, $y = 4.8(2)^x$, and $y = \frac{1}{5}(6)^x$. The computer then generates the y value 38.4. Given the three different functions, determine which one is exponential *and* produces the generated number.

36. Contests As a promotion, a clothing store draws the name of one of its customers each week. The prize is a coupon for the store. If the winner is not present at the drawing, he or she cannot claim the prize, and the amount of the coupon increases for the following week's drawing. The function $f(x) = 20(1.2)^x$ gives the amount of the coupon in dollars after x weeks of the prize going unclaimed.

> Clothes for everyone!
> **Math Apparel**
> Win Valuable Store Coupon

a. What is the amount of the coupon after 2 weeks of the prize going unclaimed?

b. After how many weeks of the prize going unclaimed will the amount of the coupon be greater than $100?

c. What is the original amount of the coupon?

d. Find the percent increase each week.

37. Critical Thinking In the definition of exponential function, the value of b cannot be 1, and the value of a cannot be 0. Why?

Graphing Calculator Graph each group of functions on the same screen. How are their graphs alike? How are they different?

38. $y = 2^x, y = 3^x, y = 4^x$

39. $y = \left(\frac{1}{2}\right)^x, y = \left(\frac{1}{3}\right)^x, y = \left(\frac{1}{4}\right)^x$

Evaluate each of the following for the given value of x.

40. $f(x) = 4^x; x = 3$

41. $f(x) = -(0.25)^x; x = 1.5$

42. $f(x) = 0.4(10)^x; x = -3$

43. This problem will prepare you for the Multi-Step Test Prep on page 796.

a. The annual tuition at a community college since 2001 is modeled by the equation $C = 2000(1.08)^n$, where C is the tuition cost and n is the number of years since 2001. What was the tuition cost in 2001?

b. What is the annual percentage of tuition increase?

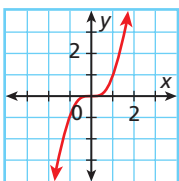**c.** Find the tuition cost in 2006.

44. Write About It Your employer offers two salary plans. With plan A, your salary is $f(x) = 10{,}000(2x)$, where x is the number of years you have worked for the company. With plan B, your salary is $g(x) = 10{,}000(2)^x$. Which plan would you choose? Why?
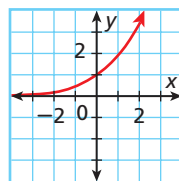
TEST PREP
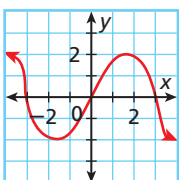
45. Which graph shows an exponential function?

Ⓐ

Ⓒ

Ⓑ

Ⓓ
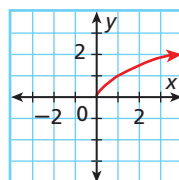
46. The function $f(x) = 15(1.4)^x$ represents the area in square inches of a photograph after it has been enlarged x times by a factor of 140%. What is the area of the photograph after it has been enlarged 4 times?

Ⓕ 5.6 square inches Ⓗ 41.16 square inches

Ⓖ 57.624 square inches Ⓙ 560 square inches

47. Look at the pattern. How many squares will there be in the nth stage?
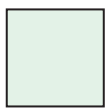
Stage 0   Stage 1   Stage 2
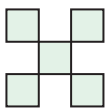
Ⓐ $5n$ Ⓑ $2.5 \cdot 2^n$ Ⓒ 25^{n-1} Ⓓ 5^n

CHALLENGE AND EXTEND

Solve each equation.

48. $4^x = 64$

49. $\left(\dfrac{1}{3}\right)^x = \dfrac{1}{27}$

50. $2^x = \dfrac{1}{16}$

51. Graph the following functions: $y = 2(2)^x$, $y = 3(2)^x$, $y = -2(2)^x$. Then make a conjecture about the relationship between the value of a and the y-intercept of $y = ab^x$.

SPIRAL REVIEW

52. The average of Roger's three test scores must be at least 90 to earn an A in his science class. Roger has scored 88 and 89 on his first two tests. Write and solve an inequality to find what he must score on the third test to earn an A. *(Lesson 3-4)*

Find the missing term in each perfect-square trinomial. *(Lesson 8-5)*

53. $x^2 + 10x + \blacksquare$

54. $4x^2 + \blacksquare + 64$

55. $\blacksquare + 42x + 49$

56. What is the 12th term of the sequence 4, 12, 36, 108, …? *(Lesson 11-1)*
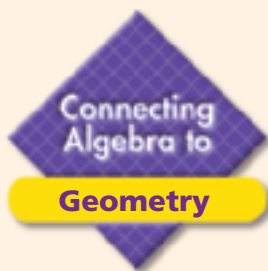
Changing Dimensions

What happens to the volume of a three-dimensional figure when you repeatedly double the dimensions?

Recall these formulas for the volumes of common three-dimensional figures.
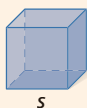
Cube $V = s^3$ **Rectangular Prism** $V = \ell wh$ **Pyramid** $V = \frac{1}{3}(\text{area of base}) \cdot h$
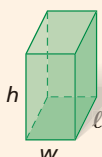
Base
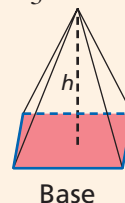
Changing the dimensions of three-dimensional figures results in geometric sequences.

Example

Find the volume of a cube with a side length of 3 cm. Double the side length and find the new volume. Repeat two more times. Show the patterns for the side lengths and volumes as geometric sequences. Identify the common ratios.

| Cube	Side Length (cm)	Volume (cm³)
1	3	27
2	6	216
3	12	1,728
4	24	13,824

× 2   × 8
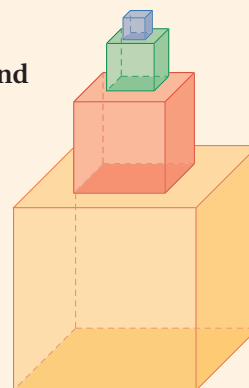
The side lengths and the volumes form geometric sequences. The sequence of the side lengths has a common ratio of **2**. The sequence of the volumes has a common ratio of 2^3, or **8**.

The patterns in the example above are a specific instance of a general rule.

> When the dimensions of a solid figure are multiplied by x, the volume of the figure is multiplied by x^3.

Try This

1. The large rectangular prism at right is 8 in. wide, 16 in. long, and 32 in. tall. The dimensions are multiplied by $\frac{1}{2}$ to create each next smaller prism. Show the patterns for the dimensions and the volumes as geometric sequences. Identify the common ratios.

2. A pyramid has a height of 8 cm and a square base of 3 cm on each edge. Triple the dimensions two times. Show the patterns for the dimensions and the volumes as geometric sequences. Identify the common ratios.

Algebra LAB
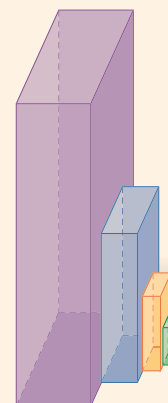
Model Growth and Decay

You can fold and cut paper to model quantities that increase or decrease exponentially.

Use with Lesson 11-3

Activity 1

1 Copy the table at right.

2 Fold a piece of notebook paper in half. Then open it back up. Count the number of regions created by the fold. Record your answer in the table.

3 Now fold the paper in half twice. Record the number of regions created by the folds in the table.

4 Repeat this process for 3, 4, and 5 folds.

Folds	Regions
0	1
1	▩
2	▩
3	▩
4	▩
5	▩

Try This

1. When the number of folds increases by 1, the number of regions ___?___ .

2. For each row of the table, write the number of regions as a power of 2.

3. Write an exponential expression for the number of regions formed by n folds.

4. If you could fold the paper 8 times, how many regions would be formed?

5. How many times would you have to fold the paper to make 512 regions?

Activity 2

1 Copy the table at right.

2 Begin with a square piece of paper. The area of the paper is 1 square unit. Cut the paper in half. Each piece has an area of $\frac{1}{2}$ square unit. Record the result in the table.

3 Cut one of those pieces in half again, and record the area of one of the new, smaller pieces in the table.

4 Repeat this process for 3, 4, and 5 cuts.

Cuts	Area
0	1
1	▩
2	▩
3	▩
4	▩
5	▩

Try This

6. When the number of cuts increases by 1, the area ___?___ .

7. For each row of the table, write the area as a power of 2.

8. Write an exponential expression for the area after n cuts.

9. What would be the area after 7 cuts?

10. How many cuts would you have to make to get an area of $\frac{1}{256}$ square unit?

11-3 Exponential Growth and Decay

Objective
Solve problems involving exponential growth and decay.

Vocabulary
exponential growth
compound interest
exponential decay
half-life

Why learn this?

Exponential growth and decay describe many real-world situations, such as the value of artwork. (See Example 1.)

Exponential growth occurs when a quantity increases by the same rate r in each time period t. When this happens, the value of the quantity at any given time can be calculated as a function of the rate and the original amount.

Know it! Note

Exponential Growth

An exponential growth function has the form $y = a(1 + r)^t$, where $a > 0$.

- y represents the final amount.
- a represents the original amount.
- r represents the rate of growth expressed as a decimal.
- t represents time.

EXAMPLE 1 Exponential Growth

The original value of a painting is $1400, and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting in 25 years.

Helpful Hint

In Example 1, round to the nearest hundredth because the problem deals with money. This means you are rounding to the nearest cent.

Step 1 Write the exponential growth function for this situation.

$y = a(1 + r)^t$ *Write the formula.*

$= 1400(1 + 0.09)^t$ *Substitute 1400 for a and 0.09 for r.*

$= 1400(1.09)^t$ *Simplify.*

Step 2 Find the value in 25 years.

$y = 1400(1.09)^t$

$= 1400(1.09)^{25}$ *Substitute 25 for t.*

$\approx 12{,}072.31$ *Use a calculator and round to the nearest hundredth.*

The value of the painting in 25 years is $12,072.31.

CHECK IT OUT!

1. A sculpture is increasing in value at a rate of 8% per year, and its value in 2000 was $1200. Write an exponential growth function to model this situation. Then find the sculpture's value in 2006.

A common application of exponential growth is *compound interest*. Recall that simple interest is earned or paid only on the principal. **Compound interest** is interest earned or paid on *both* the principal and previously earned interest.

Know it!
Note

> **Compound Interest**
>
> $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
>
> **A** represents the balance after t years.
>
> **P** represents the principal, or original amount.
>
> **r** represents the annual interest rate expressed as a decimal.
>
> **n** represents the number of times interest is compounded per year.
>
> **t** represents time in years.

EXAMPLE 2 **Finance Application**

Write a compound interest function to model each situation. Then find the balance after the given number of years.

A **$1000 invested at a rate of 3% compounded quarterly; 5 years**

Step 1 Write the compound interest function for this situation.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ *Write the formula.*

$= 1000\left(1 + \frac{0.03}{4}\right)^{4t}$ *Substitute 1000 for P, 0.03 for r, and 4 for n.*

$= 1000(1.0075)^{4t}$ *Simplify.*

Step 2 Find the balance after 5 years.

$A = 1000(1.0075)^{4(5)}$ *Substitute 5 for t.*

$= 1000(1.0075)^{20}$

≈ 1161.18 *Use a calculator and round to the nearest hundredth.*

The balance after 5 years is $1161.18.

B **$18,000 invested at a rate of 4.5% compounded annually; 6 years**

Step 1 Write the compound interest function for this situation.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ *Write the formula.*

$= 18{,}000\left(1 + \frac{0.045}{1}\right)^{t}$ *Substitute 18,000 for P, 0.045 for r, and 1 for n.*

$= 18{,}000(1.045)^{t}$ *Simplify.*

Step 2 Find the balance after 6 years.

$A = 18{,}000(1.045)^{6}$ *Substitute 6 for t.*

$\approx 23{,}440.68$ *Use a calculator and round to the nearest hundredth.*

The balance after 6 years is $23,440.68.

Reading Math

For compound interest,
- *annually* means "once per year" $(n = 1)$.
- *quarterly* means "4 times per year" $(n = 4)$.
- *monthly* means "12 times per year" $(n = 12)$.

CHECK IT OUT! Write a compound interest function to model each situation. Then find the balance after the given number of years.

2a. $1200 invested at a rate of 3.5% compounded quarterly; 4 years

2b. $4000 invested at a rate of 3% compounded monthly; 8 years

Exponential decay occurs when a quantity decreases by the same rate r in each time period t. Just like exponential growth, the value of the quantity at any given time can be calculated by using the rate and the original amount.

> ### Exponential Decay
>
> An exponential decay function has the form $y = a(1 - r)^t$, where $a > 0$.
>
> y represents the final amount.
>
> a represents the original amount.
>
> r represents the rate of decay as a decimal.
>
> t represents time.

Notice an important difference between exponential growth functions and exponential decay functions. For exponential growth, the value inside the parentheses will be greater than 1 because r is added to 1. For exponential decay, the value inside the parentheses will be less than 1 because r is subtracted from 1.

EXAMPLE 3 Exponential Decay

The population of a town is decreasing at a rate of 1% per year. In 2000 there were 1300 people. Write an exponential decay function to model this situation. Then find the population in 2008.

Step 1 Write the exponential decay function for this situation.

$y = a(1 - r)^t$ *Write the formula.*

$= 1300(1 - 0.01)^t$ *Substitute 1300 for a and 0.01 for r.*

$= 1300(0.99)^t$ *Simplify.*

Step 2 Find the population in 2008.

$y = 1300(0.99)^8$ *Substitute 8 for t.*

≈ 1200 *Use a calculator and round to the nearest whole number.*

The population in 2008 will be approximately 1200 people.

CHECK IT OUT!

3. The fish population in a local stream is decreasing at a rate of 3% per year. The original population was 48,000. Write an exponential decay function to model this situation. Then find the population after 7 years.

A common application of exponential decay is *half-life*. The **half-life** of a substance is the time it takes for one-half of the substance to decay into another substance.

> ### Half-life
>
> $A = P(0.5)^t$
>
> A represents the final amount.
>
> P represents the original amount.
>
> t represents the number of half-lives in a given time period.

EXAMPLE **4** *Science Application*

Fluorine-20 has a half-life of 11 seconds.

A **Find the amount of fluorine-20 left from a 40-gram sample after 44 seconds.**

Step 1 Find t, the number of half-lives in the given time period.

$$\frac{44 \text{ s}}{11 \text{ s}} = 4$$ *Divide the time period by the half-life.*
    The value of t is 4.

Step 2 $A = P(0.5)^t$ *Write the formula.*

$\quad\quad = 40(0.5)^4$ *Substitute 40 for P and 4 for t.*

$\quad\quad = 2.5$ *Use a calculator.*

There are 2.5 grams of fluorine-20 remaining after 44 seconds.

B **Find the amount of fluorine-20 left from a 40-gram sample after 2 minutes. Round your answer to the nearest hundredth.**

Step 1 Find t, the number of half-lives in the given time period.

$$2(60) = 120$$ *Find the number of seconds in 2 minutes.*

$$\frac{120 \text{ s}}{11 \text{ s}} = \frac{120}{11}$$ *Divide the time period by the half-life.*
    The value of t is $\frac{120}{11}$.

Step 2 $A = P(0.5)^{\frac{120}{11}}$ *Write the formula.*

$\quad\quad = 40(0.5)^8$ *Substitute 40 for P and $\frac{120}{11}$ for t.*

$\quad\quad \approx 0.02$ *Use a calculator. Round to the nearest hundredth.*

There is about 0.02 gram of fluorine-20 remaining after 2 minutes.

CHECK IT OUT!

4a. Cesium-137 has a half-life of 30 years. Find the amount of cesium-137 left from a 100-milligram sample after 180 years.

4b. Bismuth-210 has a half-life of 5 days. Find the amount of bismuth-210 left from a 100-gram sample after 5 weeks. (*Hint:* Change 5 weeks to days.)

THINK AND DISCUSS

1. Describe three real-world situations that can be described by exponential growth or exponential decay functions.

2. The population of a town after t years can be modeled by $P = 1000(1.02)^t$. Is the population increasing or decreasing? By what percentage rate?

3. An exponential function is a function of the form $y = ab^x$. Explain why both exponential growth functions and exponential decay functions are exponential functions.

Know it!
Note

4. GET ORGANIZED Copy and complete the graphic organizer.

Exponential Growth vs. Exponential Decay

How are they alike?   How are they different?

11-3 **Exercises**

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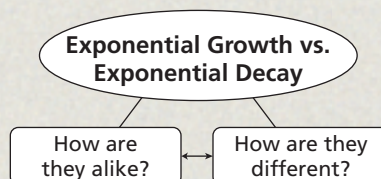
GUIDED PRACTICE

1. **Vocabulary** The function $y = 0.68(2)^x$ is an example of _____?_____.
   (*exponential growth* or *exponential decay*)

SEE EXAMPLE 1
p. 781

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

2. The cost of tuition at a college is $12,000 and is increasing at a rate of 6% per year; 4 years.

3. The number of student-athletes at a local high school is 300 and is increasing at a rate of 8% per year; 5 years.

SEE EXAMPLE 2
p. 782

Write a compound interest function to model each situation. Then find the balance after the given number of years.

4. $1500 invested at a rate of 3.5% compounded annually; 4 years

5. $4200 invested at a rate of 2.8% compounded quarterly; 6 years

SEE EXAMPLE 3
p. 783

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

6. The value of a car is $18,000 and is depreciating at a rate of 12% per year; 10 years.

7. The amount (to the nearest hundredth) of a 10-mg dose of a certain antibiotic decreases in your bloodstream at a rate of 16% per hour; 4 hours.

SEE EXAMPLE 4
p. 784

8. Bismuth-214 has a half-life of approximately 20 minutes. Find the amount of bismuth-214 left from a 30-gram sample after 1 hour.

9. Mendelevium-258 has a half-life of approximately 52 days. Find the amount of mendelevium-258 left from a 44-gram sample after 156 days.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–13	1
14–17	2
18–19	3
20	4

Extra Practice

Skills Practice p. S24
Application Practice p. S38

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

10. Annual sales for a company are $149,000 and are increasing at a rate of 6% per year; 7 years.

11. The population of a small town is 1600 and is increasing at a rate of 3% per year; 10 years.

12. A new savings account starts at $700 and increases at 1.2% quarterly; 2 years.

13. Membership of a local club grows at a rate of 7.8% every 6 months and currently has 30 members; 3 years.

Write a compound interest function to model each situation. Then find the balance after the given number of years.

14. $28,000 invested at a rate of 4% compounded annually; 5 years

15. $7000 invested at a rate of 3% compounded quarterly; 10 years

16. $3500 invested at a rate of 1.8% compounded monthly; 4 years

17. $12,000 invested at a rate of 2.6% compounded annually; 15 years

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

18. The population of a town is 18,000 and is decreasing at a rate of 2% per year; 6 years.

19. The value of a book is $58 and decreases at a rate of 10% per year; 8 years.

20. The half-life of bromine-82 is approximately 36 hours. Find the amount of bromine-82 left from an 80-gram sample after 6 days.

Identify each of the following functions as exponential growth or decay. Then give the rate of growth or decay as a percent.

21. $y = 3(1.61)^t$ **22.** $y = 39(0.098)^t$ **23.** $y = a\left(\dfrac{2}{3}\right)^t$ **24.** $y = a\left(\dfrac{3}{2}\right)^t$

25. $y = a(1.1)^t$ **26.** $y = a(0.8)^t$ **27.** $y = a\left(\dfrac{5}{4}\right)^t$ **28.** $y = a\left(\dfrac{1}{2}\right)^t$

Write an exponential growth or decay function to model each situation. Then find the value of the function after the given amount of time.

29. The population of a country is 58,000,000 and grows by 0.1% per year; 3 years.

30. An antique car is worth $32,000, and its value grows by 7% per year; 5 years.

31. An investment of $8200 loses value at a rate of 2% per year; 7 years.

32. A new car is worth $25,000, and its value decreases by 15% each year; 6 years.

33. The student enrollment in a local high school is 970 students and increases by 1.2% per year; 5 years.

34. **Archaeology** Carbon-14 dating is a way to determine the age of very old organic objects. Carbon-14 has a half-life of about 5700 years. An organic object with $\frac{1}{2}$ as much carbon-14 as its living counterpart died 5700 years ago. In 1999, archaeologists discovered the oldest bridge in England near Testwood, Hampshire. Carbon dating of the wood revealed that the bridge was 3500 years old. Suppose that when the bridge was built, the wood contained 15 grams of carbon-14. How much carbon-14 would it have contained when it was found by the archaeologists? Round to the nearest hundredth.

A computer-generated image of what the bridge at Testwood might have looked like

35. **///ERROR ANALYSIS///** Two students were asked to find the value of a $1000-item after 3 years. The item was depreciating (losing value) at a rate of 40% per year. Which is incorrect? Explain the error.

A
$1000(0.6)^3$
$216

B
$1000(0.4)^3$
$64

36. **Critical Thinking** The value of a certain car can be modeled by the function $y = 20{,}000(0.84)^t$, where t is time in years. Will the value ever be zero? Explain.

37. The value of a rare baseball card increases every year at a rate of 4%. Today, the card is worth $300. The owner expects to sell the card as soon as the value is over $600. How many years will the owner wait before selling the card? Round your answer to the nearest whole number.

38. This problem will prepare you for the Multi-Step Test Prep on page 796.

MULTI-STEP TEST PREP

  a. The annual tuition at a prestigious university was $20,000 in 2002. It generally increases at a rate of 9% each year. Write a function to describe the cost as a function of the number of years since 2002. Use 2002 as year zero when writing the function rule.

  b. What do you predict the cost of tuition will be in 2008?

  c. Use a table of values to find the first year that the cost of the tuition will be more than twice the cost in 2002.

39. Multi-Step At bank A, $600 is invested with an interest rate of 5% compounded annually. At bank B, $500 is invested with an interest rate of 6% compounded quarterly. Which account will have a larger balance after 10 years? 20 years?

40. Estimation The graph shows the decay of 100 grams of sodium-24. Use the graph to estimate the number of hours it will take the sample to decay to 10 grams. Then estimate the half-life of sodium-24.

Sodium-24

Amount remaining (g) vs *Time (h)*
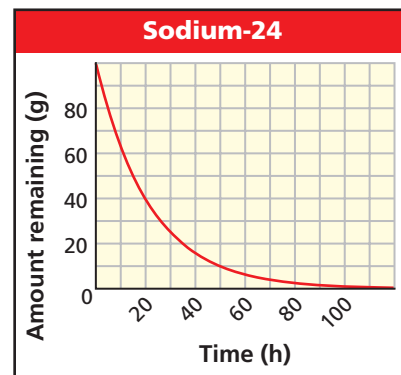
41. Graphing Calculator Use a graphing calculator to graph $y = 10(1 + r)^x$ for $r = 10\%$ and $r = 20\%$. Compare the two graphs. How does the value of r affect the graphs?

42. Write About It Write a real-world situation that could be modeled by $y = 400(1.08)^t$.

43. Write About It Write a real-world situation that could be modeled by $y = 800(0.96)^t$.

44. Critical Thinking The amount of water in a container doubles every minute. After 6 minutes, the container is full. Your friend says it was half full after 3 minutes. Do you agree? Why or why not?

TEST PREP

45. A population of 500 is decreasing by 1% per year. Which function models this situation?

  Ⓐ $y = 500(0.01)^t$ Ⓑ $y = 500(0.1)^t$ Ⓒ $y = 500(0.9)^t$ Ⓓ $y = 500(0.99)^t$

46. Which function is NOT an exponential decay model?

  Ⓕ $y = 5\left(\frac{1}{3}\right)^x$ Ⓖ $y = -5\left(\frac{1}{3}\right)^x$ Ⓗ $y = 5(3)^{-x}$ Ⓙ $y = 5(3^{-1})^x$

47. Stephanie wants to save $1000 for a down payment on a car that she wants to buy in 3 years. She opens a savings account that pays 5% interest compounded annually. About how much should Stephanie deposit now to have enough money for the down payment in 3 years?

  Ⓐ $295   Ⓑ $333   Ⓒ $500   Ⓓ $865

48. Short Response In 2000, the population of a town was 1000 and was growing at a rate of 5% per year.

  a. Write an exponential growth function to model this situation.

  b. In what year will the population be 1300? Show how you found your answer.

CHALLENGE AND EXTEND

49. You invest $700 at a rate of 6% compounded quarterly. Use a graph to estimate the number of years it will take for your investment to increase to $2300.

50. Omar invested $500 at a rate of 4% compounded annually. How long will it take for Omar's money to double? How long would it take if the interest were 8% compounded annually?

51. An 80-gram sample of a radioactive substance decayed to 10 grams after 300 minutes. Find the half-life of the substance.

52. Praseodymium-143 has a half-life of 2 weeks. The original measurement for the mass of a sample was lost. After 6 weeks, 15 grams of praseodymium-143 remain. How many grams was the original sample?

53. Phillip invested some money in a business 8 years ago. Since then, his investment has grown at an average rate of 1.3% each quarter. Phillip's investment is now worth $250,000. How much was his original investment? Round your answer to the nearest dollar.

54. **Personal Finance** Anna has a balance of $200 that she owes on her credit card. She plans to make a $30 payment each month. There is also a 1.5% finance charge (interest) on the remaining balance each month. Copy and complete the table to answer the questions below. You may add more rows to the table as necessary.

Month	Balance ($)	Monthly Payment ($)	Remaining Balance ($)	1.5% Finance Charge ($)	New Balance ($)
1	200	30	170	2.55	172.55
2	172.55	30	▨	▨	▨
3	▨	30	▨	▨	▨
4	▨	30	▨	▨	▨

   a. How many months will it take Anna to pay the entire balance?

   b. By the time Anna pays the entire balance, how much total interest will she have paid?

SPIRAL REVIEW

Write and solve a proportion for each situation. *(Lesson 2-7)*

55. A daffodil that is 1.2 feet tall casts a shadow that is 1.5 feet long. At the same time, a nearby lamppost casts a shadow that is 20 feet long. What is the height of the the lamppost?

56. A green rectangular throw pillow measures 20 inches long by 10 inches wide. A proportionally similar yellow throw pillow is 12 inches long. What is the width of the yellow pillow?

Graph each function. *(Lesson 4-4)*

57. $f(x) = 2x + 1$ **58.** $f(x) = |x - 4|$ **59.** $f(x) = x^2 - 1$

60. The function $f(x) = 0.10(2)^x$ describes the total cost in dollars of a library book fine, where x is the number of days that the book is overdue. What is the amount of the fine if a book is 4 days overdue? How many days overdue is a book if the fine is $12.80? *(Lesson 11-2)*

Linear, Quadratic, and Exponential Models

Objectives

Compare linear, quadratic, and exponential models.

Given a set of data, decide which type of function models the data and write an equation to describe the function.

Why learn this?

Different situations in sports can be described by linear, quadratic, or exponential models.

Look at the tables and graphs below. The data show three ways you have learned that variable quantities can be related. The relationships shown are linear, quadratic, and exponential.

Linear

Training Heart Rate

Age (yr)	Beats/min
20	170
30	161.5
40	153
50	144.5
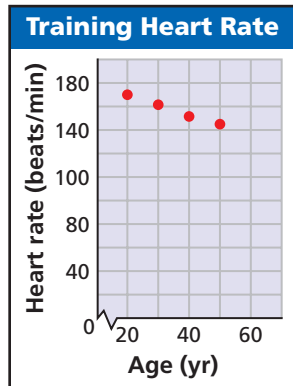
Quadratic

Volleyball Height

Time (s)	Height (ft)
0.4	10.44
0.8	12.76
1	12
1.2	9.96
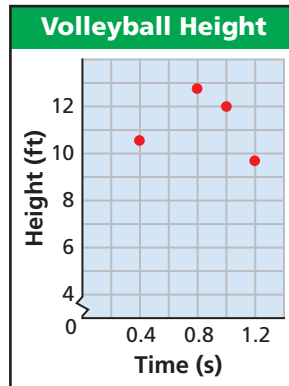
Exponential

Volleyball Tournament

Round	Teams Left
1	16
2	8
3	4
4	2

Training Heart Rate

Volleyball Height

Volleyball Tournament
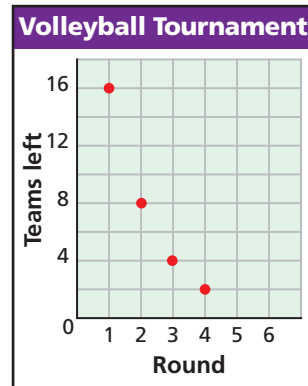
In the real world, people often gather data and then must decide what kind of relationship (if any) they think best describes their data.

EXAMPLE 1   Graphing Data to Choose a Model

Graph each data set. Which kind of model best describes the data?

A

Time (h)	0	1	2	3
Bacteria	10	20	40	80

Bacteria Population

Plot the data points and connect them.
The data appear to be exponential.
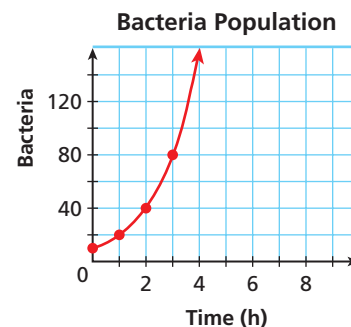
Graph each data set. Which kind of model best describes the data?

B

°C	0	5	10	15	20
°F	32	41	50	59	68

Plot the data points and connect them.
The data appear to be linear.

Celsius to Fahrenheit

Degrees Fahrenheit

Degrees Celsius
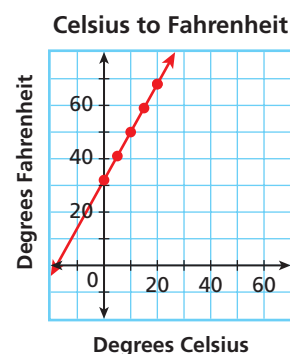
CHECK IT OUT! **Graph each data set. Which kind of model best describes the data?**

1a. $\{(-3, 0.30), (-2, 0.44), (0, 1), (1, 1.5), (2, 2.25), (3, 3.38)\}$

1b. $\{(-3, -14), (-2, -9), (-1, -6), (0, -5), (1, -6), (2, -9), (3, -14)\}$

Another way to decide which kind of relationship (if any) best describes a data set is to use patterns.

EXAMPLE 2 **Using Patterns to Choose a Model**

Look for a pattern in each data set to determine which kind of model best describes the data.

A

Height of Bridge Suspension Cables	
Cable's Distance from Tower (ft)	**Cable's Height (ft)**
0	400
100	256
200	144
300	64

+ 100, + 100, + 100 (left side)

− 144, − 112, − 80 (middle) + 32, + 32 (right)

Remember!

When the independent variable changes by a constant amount,
- linear functions have constant first differences.
- quadratic functions have constant second differences.
- exponential functions have a constant ratio.

For every constant change in distance of +100 feet, there is a constant second difference of +32.

The data appear to be quadratic.

B

Value of a Car	
Car's Age (yr)	**Value ($)**
0	20,000
1	17,000
2	14,450
3	12,282.50

+ 1, + 1, + 1 (left)   × 0.85, × 0.85, × 0.85 (right)

For every constant change in age of +1 year, there is a constant ratio of 0.85.

The data appear to be exponential.

CHECK IT OUT! 2. Look for a pattern in the data set $\{(-2, 10), (-1, 1), (0, -2), (1, 1), (2, 10)\}$ to determine which kind of model best describes the data.

After deciding which model best fits the data, you can write a function. Recall the general forms of linear, quadratic, and exponential functions.

Know it! Note

General Forms of Functions

LINEAR	QUADRATIC	EXPONENTIAL
$y = mx + b$	$y = ax^2 + bx + c$	$y = ab^x$

EXAMPLE 3 *Problem-Solving Application*

Use the data in the table to describe how the ladybug population is changing. Then write a function that models the data. Use your function to predict the ladybug population after one year.

Ladybug Population

Time (mo)	Ladybugs
0	10
1	30
2	90
3	270

1 Understand the Problem

The **answer** will have three parts—a description, a function, and a prediction.

2 Make a Plan

Determine whether the data is linear, quadratic, or exponential. Use the general form to write a function. Then use the function to find the population after one year.

3 Solve

Step 1 Describe the situation in words.

Ladybug Population

Time (mo)	Ladybugs
0	10
1	30
2	90
3	270

+ 1 ... × 3

Each month, the ladybug population is multiplied by 3. In other words, the population triples each month.

Step 2 Write the function.

There is a constant ratio of 3. The data appear to be exponential.

$y = ab^x$ *Write the general form of an exponential function.*

$y = a(3)^x$ *Substitute the constant ratio, 3, for b.*

$10 = a(3)^0$ *Choose an ordered pair from the table, such as (0, 10). Substitute for x and y.*

$10 = a(1)$ *Simplify. $3^0 = 1$*

$10 = a$ *The value of a is 10.*

$y = 10(3)^x$ *Substitute 10 for a in $y = a(3)^x$.*

Helpful Hint

You can choose any given ordered pair to substitute for x and y. However, it is often easiest to choose an ordered pair that contains 0.

Step 3 Predict the ladybug population after one year.

$y = 10(3)^x$ *Write the function.*

$\quad = 10(3)^{12}$ *Substitute 12 for x (1 year = 12 mo).*

$\quad = 5{,}314{,}410$ *Use a calculator.*

There will be 5,314,410 ladybugs after one year.

4 Look Back

You chose the ordered pair $(0, 10)$ to write the function. Check that every other ordered pair in the table satisfies your function.

$y = 10(3)^x$	
30	$10(3)^1$
30	$10(3)$
30	30 ✓

$y = 10(3)^x$	
90	$10(3)^2$
90	$10(9)$
90	90 ✓

$y = 10(3)^x$	
270	$10(3)^3$
270	$10(27)$
270	270 ✓

CHECK IT OUT!

3. Use the data in the table to describe how the oven temperature is changing. Then write a function that models the data. Use your function to predict the temperature after 1 hour.

Oven Temperature				
Time (min)	0	10	20	30
Temperature (°F)	375	325	275	225

Student to Student

Checking Units

Michael Gambhir
Warren High School

I used to get a lot of answers wrong because of the units. If a question asked for the value of something after 1 year, I would always just substitute 1 into the function.

I finally figured out that you have to check what x is. If x represents months and you're trying to find the value after 1 year, then you have to substitute 12, not 1, because there are 12 months in a year.

THINK AND DISCUSS

1. Do you think that every data set will be able to be modeled by a linear, quadratic, or exponential function? Why or why not?

2. In Example 3, is it certain that there will be 5,314,410 ladybugs after one year? Explain.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, list some characteristics and sketch a graph of each type of model.

Modeling Data
- Linear
- Quadratic
- Exponential

11-4 **Exercises**

go.hrw.com
Homework Help Online
KEYWORD: MA7 11-4
Parent Resources Online
KEYWORD: MA7 Parent
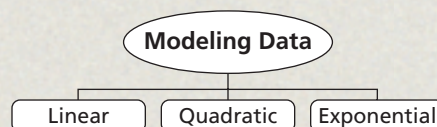
GUIDED PRACTICE

SEE EXAMPLE 1
p. 789

Graph each data set. Which kind of model best describes the data?

1. $\{(-1, 4), (-2, 0.8), (0, 20), (1, 100), (-3, 0.16)\}$

2. $\{(0, 3), (1, 9), (2, 11), (3, 9), (4, 3)\}$

3. $\{(2, -7), (-2, -9), (0, -8), (4, -6), (6, -5)\}$

SEE EXAMPLE 2
p. 790

Look for a pattern in each data set to determine which kind of model best describes the data.

4. $\{(-2, 1), (-1, 2.5), (0, 3), (1, 2.5), (2, 1)\}$

5. $\{(-2, 0.75), (-1, 1.5), (0, 3), (1, 6), (2, 12)\}$

6. $\{(-2, 2), (-1, 4), (0, 6), (1, 8), (2, 10)\}$

SEE EXAMPLE 3
p. 791

7. Consumer Economics Use the data in the table to describe the cost of grapes. Then write a function that models the data. Use your function to predict the cost of 6 pounds of grapes.

Total Cost of Grapes				
Amount (lb)	1	2	3	4
Cost ($)	1.79	3.58	5.37	7.16

PRACTICE AND PROBLEM SOLVING

Graph each data set. Which kind of model best describes the data?

8. $\{(-3, -5), (-2, -8), (-1, -9), (0, -8), (1, -5), (2, 0), (3, 7)\}$

9. $\{(-3, -1), (-2, 0), (-1, 1), (0, 2), (1, 3), (2, 4), (3, 5)\}$

10. $\{(0, 0.1), (2, 0.9), (3, 2.7), (4, 8.1)\}$

Look for a pattern in each data set to determine which kind of model best describes the data.

11. $\{(-2, 5), (-1, 4), (0, 3), (1, 2), (2, 1)\}$

12. $\{(-2, 12), (-1, 15), (0, 16), (1, 15), (2, 12)\}$

13. $\{(-2, 8), (-1, 4), (0, 2), (1, 1), (2, 0.5)\}$

14. Business Use the data in the table to describe how the company's sales are changing. Then write a function that models the data. Use your function to predict the amount of sales after 10 years.

Company Sales				
Year	0	1	2	3
Sales ($)	25,000	30,000	36,000	43,200

15. Multi-Step Jay's hair grows about 6 inches each year. Write a function that describes the length ℓ in inches that Jay's hair will grow for each year k. Which kind of model best describes the function?

Which kind of model best describes each situation?

16. The height of a plant at weekly intervals over the last 6 weeks was 1 inches, 1.5 inches, 2 inches, 2.5 inches, 3 inches., and 3.5 inches.

17. The number of games a baseball player played in the last four years was 162, 162, 162, and 162.

18. The height of a ball in a certain time interval was recorded as 30.64 feet, 30.96 feet, 31 feet, 30.96 feet, and 30.64 feet.

Write a function to model each set of data.

19.

x	−1	0	1	2	4
y	0.05	0.2	0.8	3.2	51.2

20.

x	−2	0	2	4	8
y	5	4	3	2	0

Which kind of model best describes each graph?

21.

Cost of a Telephone Call

22.

Height of Basketball
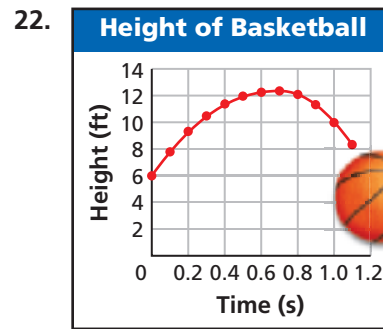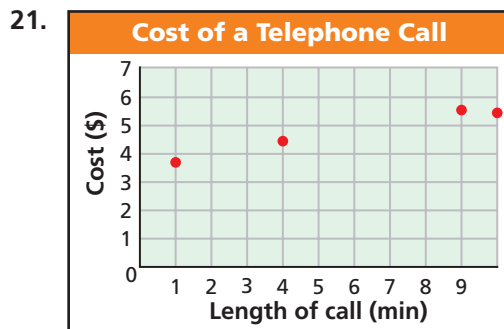
23. Write About It Write a set of data that you could model with an exponential function. Explain why the exponential model would work.

24. ///ERROR ANALYSIS/// A student concluded that the data set would best be modeled by a quadratic function. Explain the student's error.

25. Critical Thinking Sometimes the graphs of quadratic data and exponential data can look very similar. Describe how you can tell them apart.

MULTI-STEP TEST PREP

26. This problem will prepare you for the Multi-Step Test Prep on page 796.

   a. Examine the two models that represent annual tuition for two colleges. Describe each model as linear, quadratic, or exponential.

   b. Write a function rule for each model.

   c. Both models have the same values for year 0. What does this mean?

   d. Why do both models have the same value for year 1?

Years After 2004	Tuition at College 1 ($)	Tuition at College 2 ($)
0	2000.00	2000.00
1	2200.00	2200.00
2	2400.00	2420.00
3	2600.00	2662.00
4	2800.00	2928.20

27. Which function best models the data: $\{(-4, -2), (-2, -1), (0, 0), (2, 1), (4, 2)\}$?

   (A) $y = \left(\frac{1}{2}\right)^x$ (B) $y = \frac{1}{2}x^2$ (C) $y = \frac{1}{2}x$ (D) $y = \left(\frac{1}{2}x\right)^2$

28. A city's population is increasing at a rate of 2% per year. Which type of model describes this situation?

   (F) Exponential   (G) Quadratic   (H) Linear   (J) None of these

29. Which data set is best modeled by a linear function?

   (A) $\{(-2, 0), (-1, 2), (0, -4), (1, -1), (2, 2)\}$

   (B) $\{(-2, 2), (-1, 4), (0, 6), (1, 16), (2, 32)\}$

   (C) $\{(-2, 2), (-1, 4), (0, 6), (1, 8), (2, 10)\}$

   (D) $\{(-2, 0), (-1, 5), (0, 7), (1, 5), (2, 0)\}$

CHALLENGE AND EXTEND

30. **Finance** An accountant estimates that a certain new automobile worth $18,000 will lose value at a rate of 16% per year.

   a. Make a table that shows the worth of the car for years 0, 1, 2, 3, and 4. What is the real-world meaning of year 0?

   b. Which type of model best represents the data in your table? Explain.

   c. Write a function for your data.

   d. What is the value of the car after $5\frac{1}{2}$ years?

   e. What is the value of the car after 8 years?

31. **Pet Care** The table shows general guidelines for the weight of a Great Dane at various ages.

   a. None of the three models in this lesson—linear, quadratic, or exponential—fits this data exactly. Which of these is the *best* model for the data? Explain your choice.

   b. What would you predict for the weight of a Great Dane who is 1 year old?

   c. Do you think you could use your model to find the weight of a Great Dane at any age? Why or why not?

Great Dane	
Age (mo)	Weight (kg)
2	12
4	23
6	33
8	40
10	45

SPIRAL REVIEW

Write an algebraic expression for each situation. *(Lesson 1-1)*

32. the total number of kilometers run by Helen in n 5-kilometer races

33. the average gas mileage of a car that travels 145 miles on g gallons of gasoline

34. Lorraine's height if she is b inches shorter than Gene, who is 74 inches tall

Solve by using square roots. *(Lesson 9-7)*

35. $4x^2 = 100$ 36. $10 - x^2 = 10$ 37. $16x^2 + 5 = 86$

Graph each exponential function. *(Lesson 11-2)*

38. $y = 6^x$ 39. $y = -2(5)^x$ 40. $y = \left(\frac{1}{3}\right)^x$

MULTI-STEP TEST PREP

Exponential Functions

Dollars for Scholars In 1980, the average annual tuition at two-year colleges was $350. Since then, the cost of tuition has increased by an average of 9% each year.

1. Write a function rule that models the annual growth in tuition at two-year colleges since 1980. Let 1980 be year zero in your function. Identify the variables, and tell which is independent and which is dependent.

2. Use your function to determine the average annual tuition in 2006. Use a table and a graph to support your answer.

3. Use your function to predict the average annual tuition at two-year colleges for the year you plan to graduate from high school.

4. In what year is the average annual tuition twice as much as in 1980? Use a table and a graph to support your answer.

5. In what year does the average annual tuition reach $1000? Use a table and a graph to support your answer.

READY TO GO ON?

Quiz for Lessons 11-1 Through 11-4

11-1 Geometric Sequences

Find the next three terms in each geometric sequence.

1. 3, 6, 12, 24, …   **2.** −1, 2, −4, 8, …   **3.** −2400, −1200, −600, −300, …

4. The first term of a geometric sequence is 2 and the common ratio is 3. What is the 8th term of the sequence?

5. The table shows the distance swung by a pendulum during its first three swings. The values form a geometric sequence. What will be the length of the 7th swing?

Swing	Length (cm)
1	1000
2	800
3	640

11-2 Exponential Functions

6. The function $f(x) = 3(1.1)^x$ gives the length (in inches) of an image after being enlarged by 10% x times. What is the length of the image after it has been enlarged 4 times? Round your answer to the nearest hundredth.

Graph each exponential function.

7. $y = 3^x$ **8.** $y = 2(2)^x$ **9.** $y = -2(4)^x$ **10.** $y = -(0.5)^x$

11. The function $f(x) = 40(0.8)^x$ gives the amount of a medication in milligrams present in a patient's system x hours after taking a 40-mg dose. In how many hours will there be less than 2 mg of the drug in a patient's system?

11-3 Exponential Growth and Decay

Write a function to model each situation. Then find the value of the function after the given amount of time.

12. Fiona's salary is $30,000, and she expects to receive a 3% raise each year; 10 years.

13. $2000 is invested at a rate of 4.5% compounded monthly; 3 years.

14. A $1200 computer is losing value at a rate of 20% per year; 4 years.

15. Strontium-90 has a half-life of 29 years. About how much strontium-90 will be left from a 100-mg sample after 290 years? Round your answer to the nearest thousandth.

11-4 Linear, Quadratic, and Exponential Models

Graph each data set. Which kind of model best describes the data?

16. $\{(-2, 5), (3, 10), (0, 1), (1, 2), (0.5, 1.25)\}$ **17.** $\{(0, 3), (2, 12), (-1, 1.5), (-3, 0.375), (4, 48)\}$

Look for a pattern in each data set to determine which kind of model best describes the data.

18. $\{(-2, -6), (-1, -5), (0, -4), (1, -3), (2, -2)\}$ **19.** $\{(-2, -24), (-1, -12), (0, -6), (1, -3)\}$

20. Use the data in the table to describe how the value of the stamp is changing. Then write a function that models the data. Use your function to predict the value of the stamp in 11 years.

Value of Collectible Stamp				
Year	0	1	2	3
Value ($)	5.00	6.00	7.20	8.64

11-5 Square-Root Functions

Who uses this?
Astronauts can use square-root functions to calculate their speed in free fall.

Astronauts at NASA practice living in the weightlessness of space by training in the KC-135, also known as the "Vomit Comet." This aircraft flies to a certain altitude and then free falls for a period of time, simulating a zero-gravity environment.

The function $y = 8\sqrt{x}$ gives the speed in feet per second of an object in free fall after falling x feet. This function is different from others you have seen so far. It contains a variable under the square-root sign, $\sqrt{}$.

Know it! Note — Square-Root Function

WORDS	EXAMPLES	NONEXAMPLES
A **square-root function** is a function whose rule contains a variable under a square-root sign.	$y = \sqrt{x}$ $y = \sqrt{2x + 1}$ $y = 3\sqrt{\dfrac{x}{2}} - 6$	$y = x^2$ $y = \dfrac{2}{x + 1}$ $y = \sqrt{3}x$

EXAMPLE 1 Evaluating Square-Root Functions

A Find the speed of an object in free fall after it has fallen 4 feet.

$y = 8\sqrt{x}$ *Write the speed function.*
$ = 8\sqrt{4}$ *Substitute 4 for x.*
$ = 8(2)$ *Simplify.*
$ = 16$

After an object has fallen 4 feet, its speed is 16 ft/s.

Helpful Hint
Check that your answer is reasonable. In Example 1B, $8\sqrt{49} = 8(7) = 56$, so $8\sqrt{50} \approx 56.6$ is reasonable.

B Find the speed of an object in free fall after it has fallen 50 feet. Round your answer to the nearest tenth.

$y = 8\sqrt{x}$ *Write the speed function.*
$ = 8\sqrt{50}$ *Substitute 50 for x.*
$ \approx 56.6$ *Use a calculator.*

After an object has fallen 50 feet, its speed is about 56.6 ft/s.

CHECK IT OUT!
1a. Find the speed of an object in free fall after it has fallen 25 feet.
1b. Find the speed of an object in free fall after it has fallen 15 feet. Round your answer to the nearest hundredth.

Recall that the square root of a negative number is not a real number. The domain (*x*-values) of a square-root function is restricted to numbers that make the value under the radical sign greater than or equal to 0.

EXAMPLE 2 **Finding the Domain of Square-Root Functions**

Find the domain of each square-root function.

A $y = \sqrt{x + 4} - 3$

$x + 4 \geq 0$ *The expression under the radical sign must be greater than or equal to 0.*

$\dfrac{-4 \quad -4}{x \quad \geq -4}$ *Solve the inequality. Subtract 4 from both sides.*

The domain is the set of all real numbers greater than or equal to -4.

B $y = \sqrt{3(x - 2)}$

$3(x - 2) \geq 0$ *The expression under the radical sign must be greater than or equal to 0.*

$3x - 6 \geq 0$ *Solve the inequality. Distribute 3 on the left side.*

$\dfrac{+6 \quad +6}{3x \quad \geq \quad 6}$ *Add 6 to both sides.*

$x \geq 2$ *Divide both sides by 3.*

The domain is the set of all real numbers greater than or equal to 2.

CHECK IT OUT! Find the domain of each square-root function.

2a. $y = \sqrt{2x - 1}$ **2b.** $y = \sqrt{3x - 5}$

The parent function for square-root functions, $f(x) = \sqrt{x}$, is graphed at right. Notice there are no *x*-values to the left of 0 because the domain is $x \geq 0$.

Know it! Note
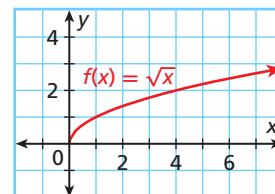
Translations of the Graph of $f(x) = \sqrt{x}$

The graph of $f(x) = \sqrt{x} + c$ is a vertical translation of the graph of $f(x) = \sqrt{x}$.	The graph of $f(x) = \sqrt{x - a}$ is a horizontal translation of the graph of $f(x) = \sqrt{x}$.
The graph is translated *c* units up for $c > 0$ and *c* units down for $c < 0$.	The graph is translated *a* units right for $a > 0$ and *a* units left for $a < 0$.
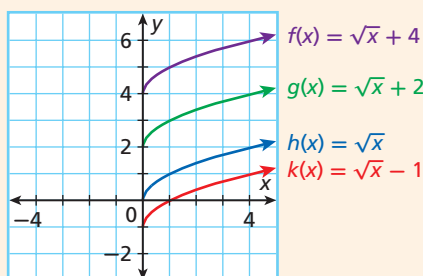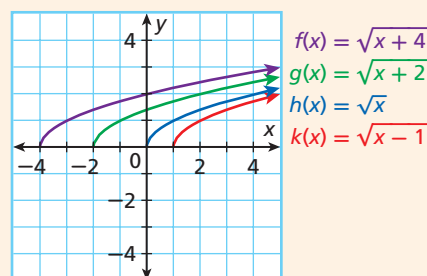
If a square-root function is given in one of these forms, you can graph the parent function $f(x) = \sqrt{x}$ and translate it vertically or horizontally.

11-5 Square-Root Functions   **799**

EXAMPLE **3** **Graphing Square-Root Functions**

A Graph $f(x) = \sqrt{x - 4}$.
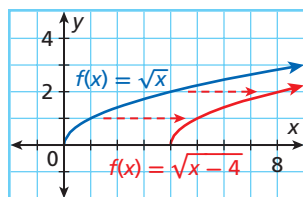
Since this function is in the form $f(x) = \sqrt{x - a}$, you can graph it as a horizontal translation of the graph of $f(x) = \sqrt{x}$.

Graph $f(x) = \sqrt{x}$ and then shift the graph 4 units to the right.

B Graph $f(x) = \sqrt{2x} + 3$.

This is not a horizontal or vertical translation of the graph of $f(x) = \sqrt{x}$.

Step 1 Find the domain of the function.

$2x \geq 0$ *The expression under the radical sign must be greater than or equal to 0.*

$x \geq 0$ *Solve the inequality by dividing both sides by 2.*

The domain is the set of all real numbers greater than or equal to 0.

Step 2 Choose x-values greater than or equal to 0 and generate ordered pairs.

x	$f(x) = \sqrt{2x} + 3$
0	3
2	5
8	7
18	9
32	11

Step 3 Plot the points. Then connect them with a smooth curve.

> **Helpful Hint**
>
> In Example 3B, when generating ordered pairs, choose x-values that make the expression under the radical sign a perfect square.
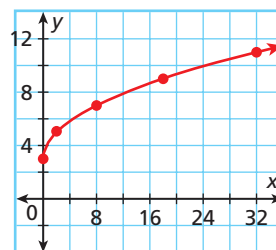
CHECK IT OUT! Graph each square-root function.

3a. $f(x) = \sqrt{x} + 2$ **3b.** $f(x) = 2\sqrt{x} + 3$

THINK AND DISCUSS

1. How do you find the domain of a square-root function?

2. Compare the graph of $f(x) = \sqrt{x + 8}$ with the graph of $f(x) = \sqrt{x}$.

3. Compare the graph of $f(x) = \sqrt{x + 8}$ with the graph of $f(x) = \sqrt{x} + 8$.

4. GET ORGANIZED Copy and complete the graphic organizer. In each box, graph the function and give its domain.

Square-Root Functions

| $y = \sqrt{x}$ | $y = \sqrt{x} + 5$ | $y = \sqrt{x + 5}$ | $y = \sqrt{5x}$ |

11-5
Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 11-5
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain why $y = x + \sqrt{3}$ is not a *square-root function*.

SEE EXAMPLE 1
p. 798

2. **Geometry** In a right triangle, $c = \sqrt{a^2 + b^2}$, where c is the length of the hypotenuse (the longest side) and a and b are the lengths of the other two sides, called the legs. What is the length of the hypotenuse of a right triangle if its legs measure 14 cm and 8 cm? Round your answer to the nearest hundredth.
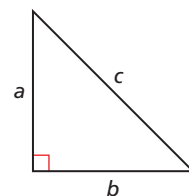
SEE EXAMPLE 2
p. 799

Find the domain of each square-root function.

3. $y = \sqrt{x + 6}$
4. $y = 4 - \sqrt{3 - x}$
5. $y = \sqrt{2x} - 5$
6. $y = \sqrt{x + 2}$
7. $y = \sqrt{3x + 9}$
8. $y = x + \sqrt{x - 5}$

SEE EXAMPLE 3
p. 800

Graph each square-root function.

9. $f(x) = \sqrt{x - 1}$
10. $f(x) = -\sqrt{2x}$
11. $f(x) = \sqrt{x} + 1$
12. $f(x) = \sqrt{x} - 12$
13. $f(x) = \sqrt{4 - x}$
14. $f(x) = \sqrt{x + 4}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15	1
16–27	2
28–33	3

15. **Law Enforcement** At the scene of a car accident, police measure the length of the skid marks to estimate the speed that the car was traveling. On dry concrete, $f(x) = \sqrt{24x}$ gives the speed in mi/h when the length of the skid mark is x feet. Find the speed that a car was traveling if it left a skid mark that was 104 ft long. Round your answer to the nearest hundredth.

Extra Practice
Skills Practice p. S25
Application Practice p. S38

Find the domain of each square-root function.

16. $y = \sqrt{8 - 2x}$
17. $y = 4 - \sqrt{\dfrac{x}{2}}$
18. $y = \sqrt{3x + 2}$

19. $y = \sqrt{-2x + 3}$
20. $y = 2\sqrt{x + 1} - 2$
21. $y = \sqrt{3(x + 2) - 1}$

22. $y = \sqrt{2(x + 4)} - 3$
23. $y = 7\sqrt{\dfrac{x}{5} - 8}$
24. $y = \sqrt{2(3x - 6)}$

25. $y = \sqrt{\dfrac{1}{3}(x - 9)}$
26. $y = \sqrt{2(x + 7) - 6}$
27. $y = 4 + \sqrt{3x + 2}$

Graph each square-root function.

28. $f(x) = \sqrt{x - 5}$
29. $f(x) = \sqrt{2x} - 4$
30. $f(x) = -1 - \sqrt{x}$

31. $f(x) = \sqrt{x} - 4$
32. $f(x) = 3\sqrt{x - 6}$
33. $f(x) = \dfrac{1}{2}\sqrt{x + 4}$

34. **Geometry** If you know a circle's area, you can use the formula $r = \sqrt{\dfrac{A}{\pi}}$ to find the radius. What is the radius of a circle whose area is 60 cm²? Use 3.14 for π. Round your answer to the nearest hundredth of a centimeter.

35. **Graphing Calculator** Use a graphing calculator for the following.
    a. Graph $y = \sqrt{x}$, $y = \dfrac{1}{2}\sqrt{x}$, $y = 2\sqrt{x}$, $y = 3\sqrt{x}$, and $y = 4\sqrt{x}$ on the same screen.
    b. What is the domain of each function?
    c. What is the range of each function?
    d. Describe the characteristics of $y = a\sqrt{x}$ for $a > 0$.

36. Graphing Calculator Use a graphing calculator for the following.

    a. Graph $y = -\sqrt{x}$, $y = -\frac{1}{2}\sqrt{x}$, $y = -2\sqrt{x}$, $y = -3\sqrt{x}$, and $y = -4\sqrt{x}$ on the same screen.

    b. What is the domain of each function?

    c. What is the range of each function?

    d. Describe the characteristics of $y = a\sqrt{x}$ for $a < 0$.

37. The distance d between two points (x, y) and (w, z) in the coordinate plane can be found by using the formula $d = \sqrt{(w - x)^2 + (z - y)^2}$. What is the distance between the points $(2, 1)$ and $(5, 3)$? Round your answer to the nearest hundredth.

38. Geology Tsunamis are large waves that move across deep oceans at high speeds. When tsunamis hit shallow water, their energy moves them upward into a destructive force. The speed of a tsunami in meters per second can be found using the function $f(x) = \sqrt{9.8x}$, where x is the depth of the water in meters. Graph this function. Then find the speed of a tsunami when the water depth is 500 meters.

39. Astronomy A planet's *escape velocity* is the initial velocity that an object must have to escape the planet's gravity. Escape velocity v in meters per second can be found by using the formula $v = \sqrt{2gr}$, where g is the planet's surface gravity and r is the planet's radius. Find the escape velocity for each planet in the table. Round your answers to the nearest whole number.

Planet	g (m/s²)	r (m)
Mercury	3.7	2.4×10^6
Venus	8.8	6.1×10^6
Earth	9.8	6.4×10^6
Mars	3.7	3.4×10^6

40. Estimation The volume V of a cylinder can be found by using the formula $V = \pi r^2 h$, where r represents the radius of the cylinder and h represents its height. Find the radius of a cylinder whose volume is 1212 in³ and whose height is 10 inches. Use 3.14 for π.
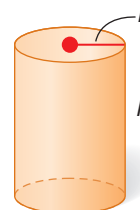
41. Write About It Explain how to find the domain of a square-root function. Why is the domain not all real numbers?

42. Multi-Step For the function $y = \sqrt{3(x - 5)}$, find the value of y that corresponds to the least possible value for x.

43. Critical Thinking Can the range of a square-root function be all real numbers? Explain.

MULTI-STEP TEST PREP

44. This problem will prepare you for the Multi-Step Test Prep on page 830.

    a. The Ocean Motion ride at Ohio's Cedar Point amusement park is a giant ship that swings like a pendulum. If a pendulum is under the influence of gravity only, then the time in seconds that it takes for one complete swing back and forth (called the pendulum's period) is $T = 2\pi\sqrt{\frac{\ell}{32}}$, where ℓ is the length of the pendulum in feet. What is the domain of this function?

    b. What is the period of a pendulum whose length is 80 feet? Use 3.14 for π and round your answer to the nearest hundredth.

    c. The length of the Ocean Motion pendulum is about 80 feet. Do you think your answer to part **b** is its period? Explain why or why not.

45. Which function is graphed at right?

(A) $f(x) = \sqrt{x + 3}$ (C) $f(x) = \sqrt{x - 3}$

(B) $f(x) = \sqrt{x} + 3$ (D) $f(x) = \sqrt{x} - 3$
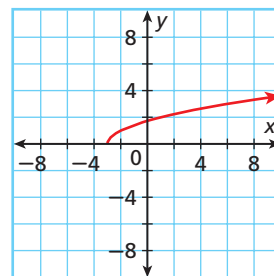
46. Which function has domain $x \geq 2$?

(F) $y = \sqrt{2x}$ (H) $y = \sqrt{\dfrac{x}{2}}$

(G) $y = \sqrt{x + 2}$ (J) $y = \sqrt{x - 2}$

47. The function $y = \sqrt{\frac{1}{5}x}$ gives the approximate time y in seconds that it takes an object to fall to the ground from a height of x meters. About how long will it take an object 25 meters above the ground to fall to the ground?

(A) 11.2 seconds   (C) 2.2 seconds

(B) 5 seconds   (D) 0.4 seconds

48. Gridded Response If $g(x) = \sqrt{4x} - 1$, what is $g(9)$?

CHALLENGE AND EXTEND

Find the domain of each function.

49. $y = \sqrt{x^2 - 25}$

50. $y = \sqrt{x^2 + 5x + 6}$

51. $y = \sqrt{2x^2 + 5x - 12}$

Find the domain and range of each function.

52. $y = 2 - \sqrt{x + 3}$

53. $y = 4 - \sqrt{3 - x}$

54. $y = 6 - \sqrt{\dfrac{x}{2}}$

55. Give an example of a square-root function whose graph is above the x-axis.

56. Give an example of a square-root function whose graph is in Quadrant IV.

57. Multi-Step Justin is given the function $y = 3 - \sqrt{2(x - 5)}$ and $x = 2, 4, 5,$ and 7. He notices that two of these values are not in the function's domain.

  a. Which two values are not in the domain? How do you know?

  b. What are the values of y for the two given x-values that are in the domain?

SPIRAL REVIEW

Write each equation in slope-intercept form, and then graph. *(Lesson 5-6)*

58. $2y = 4x - 8$

59. $3x + 6y = 12$

60. $2x = -y - 9$

Find each product. *(Lesson 7-8)*

61. $(3x - 1)^2$

62. $(2x - 5)(2x + 5)$

63. $(a - b^2c)^2$

64. $(x^2 + 2y)^2$

65. $(3r - 2s)(3r + 2s)$

66. $(a^3b^2 - c^4)(a^3b^2 + c^4)$

67. Blake invested $42,000 at a rate of 5% compounded quarterly. Write a function to model this situation. Then find the value of Blake's investment after 3 years. *(Lesson 11-3)*

68. Lead-209 has a half-life of about 3.25 hours. Find the amount of lead-209 left from a 230-mg sample after 1 day. Round your answer to the nearest hundredth. *(Lesson 11-3)*

Technology LAB

Graph Radical Functions

You can use a graphing calculator to graph radical functions or to quickly check that a graph drawn by hand is reasonable.

Use with Lesson 11-5

Activity

Graph $f(x) = \sqrt{x-1}$ without using a graphing calculator. Then graph the same function on a graphing calculator and compare.

1 The graph will be a shift of the graph of the parent function $f(x) = \sqrt{x}$ one unit right. Graph $f(x) = \sqrt{x}$ and then shift the graph one unit right.
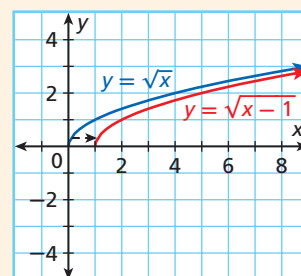
2 Enter the function into the **Y=** editor.

| Y= | 2nd | $\sqrt{}$ x^2 | (| X,T,θ,*n* | − | 1 |) |

Press **WINDOW** and enter -1 for **Xmin**, 9 for **Xmax**, -5 for **Ymin**, and 5 for **Ymax**.
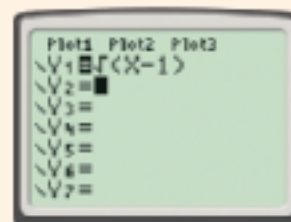
Press **GRAPH** and compare the graph on the screen to the graph done by hand.

The graph on the screen indicates that the graph done by hand is reasonable.
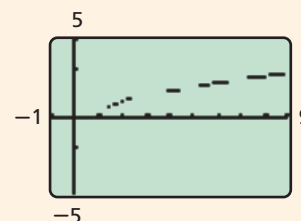
Try This

1. Graph $f(x) = \sqrt{x+3}$ without using a graphing calculator. Then graph the same function on a graphing calculator and compare.

2. Graph $f(x) = \sqrt{x} - 2$ without using a graphing calculator. Then graph the same function on a graphing calculator and compare.

3. **Make a Conjecture** How do you think the graph of $f(x) = \sqrt{x+1} + 4$ compares to the graph of $f(x) = \sqrt{x}$? Use a graphing calculator to check your conjecture.

4. **Make a Conjecture** How do you think the graph of $f(x) = 2\sqrt{x}$ compares to the graph of $f(x) = \sqrt{x}$? Use a graphing calculator to check your conjecture.

11-6 Radical Expressions

Vocabulary
radical expression
radicand

Why learn this?
You can use a radical expression to find the length of a throw in baseball. (See Example 5.)
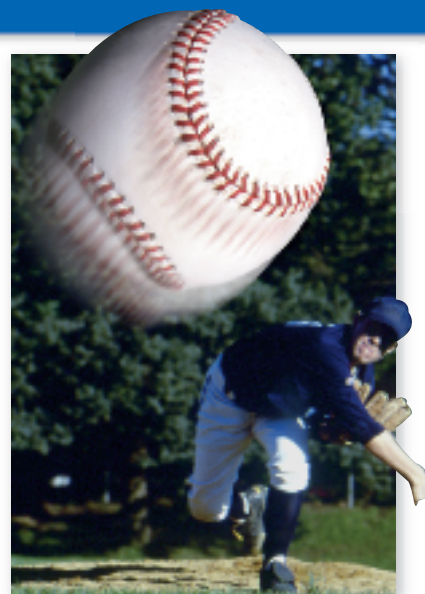
An expression that contains a radical sign $\left(\sqrt{}\right)$ is a **radical expression**. There are many different types of radical expressions, but in this course, you will only study radical expressions that contain square roots.

Examples of radical expressions:

$$\sqrt{14} \quad \sqrt{\ell^2 + w^2} \quad \sqrt{2gd} \quad \frac{\sqrt{d}}{4} \quad 5\sqrt{2} \quad \sqrt{18}$$

The expression under a radical sign is the **radicand**. A radicand may contain numbers, variables, or both. It may contain one term or more than one term.

Simplest Form of a Square-Root Expression

An expression containing square roots is in simplest form when
- the radicand has no perfect square factors other than 1.
- the radicand has no fractions.
- there are no square roots in any denominator.

Remember that positive numbers have two square roots, one positive and one negative. However, $\sqrt{}$ indicates a nonnegative square root. When you simplify, be sure that your answer is not negative. To simplify $\sqrt{x^2}$, you should write $\sqrt{x^2} = |x|$, because you do not know whether x is positive or negative.

Below are some simplified square-root expressions:

$$\sqrt{x^2} = |x| \qquad \sqrt{x^3} = x\sqrt{x} \qquad \sqrt{x^4} = x^2 \qquad \sqrt{x^5} = x^2\sqrt{x} \qquad \sqrt{x^6} = |x^3|$$

EXAMPLE 1 Simplifying Square-Root Expressions

Simplify each expression.

A $\sqrt{\dfrac{2}{72}}$

$\sqrt{\dfrac{2}{72}} = \sqrt{\dfrac{1}{36}}$

$= \dfrac{1}{6}$

B $\sqrt{3^2 + 4^2}$

$\sqrt{3^2 + 4^2} = \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

C $\sqrt{x^2 + 8x + 16}$

$\sqrt{x^2 + 8x + 16} = \sqrt{(x+4)^2}$

$= |x + 4|$

CHECK IT OUT! Simplify each expression.

1a. $\sqrt{\dfrac{256}{4}}$ **1b.** $\sqrt{40 + 9}$ **1c.** $\sqrt{5^2 + 12^2}$ **1d.** $\sqrt{(3 - x)^2}$

Product Property of Square Roots

WORDS	NUMBERS	ALGEBRA
For any nonnegative real numbers a and b, the square root of ab is equal to the square root of a times the square root of b.	$\sqrt{4(25)} = \sqrt{100} = 10$ $\sqrt{4(25)} = \sqrt{4}\sqrt{25} = 2(5) = 10$	$\sqrt{ab} = \sqrt{a}\sqrt{b}$, where $a \geq 0$ and $b \geq 0$

EXAMPLE 2 **Using the Product Property of Square Roots**

Simplify. All variables represent nonnegative numbers.

A $\sqrt{18}$

$\begin{aligned}\sqrt{18} &= \sqrt{9(2)} && \textit{Factor the radicand using perfect squares.}\\ &= \sqrt{9}\sqrt{2} && \textit{Product Property of Square Roots}\\ &= 3\sqrt{2} && \textit{Simplify.}\end{aligned}$

When factoring the radicand, use factors that are perfect squares. In Example 2A, you could have factored 18 as 6 · 3, but this contains no perfect squares.

B $\sqrt{x^4 y^3}$

$\begin{aligned}\sqrt{x^4 y^3} &= \sqrt{x^4}\sqrt{y^3} && \textit{Product Property of Square Roots}\\ &= \sqrt{x^4}\sqrt{y^2}\sqrt{y} && \textit{Product Property of Square Roots}\\ &= x^2 y\sqrt{y} && \textit{Since } y \text{ is nonnegative, } \sqrt{y^2} = y.\end{aligned}$

CHECK IT OUT!

Simplify. All variables represent nonnegative numbers.

2a. $\sqrt{128}$ **2b.** $\sqrt{x^3 y^2}$ **2c.** $\sqrt{48 a^2 b}$

Quotient Property of Square Roots

WORDS	NUMBERS	ALGEBRA
For any real numbers a and b ($a \geq 0$ and $b > 0$), the square root of $\frac{a}{b}$ is equal to the square root of a divided by the square root of b.	$\sqrt{\dfrac{36}{4}} = \sqrt{9} = 3$ $\sqrt{\dfrac{36}{4}} = \dfrac{\sqrt{36}}{\sqrt{4}} = \dfrac{6}{2} = 3$	$\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

EXAMPLE 3 **Using the Quotient Property of Square Roots**

Simplify. All variables represent nonnegative numbers.

A $\sqrt{\dfrac{5}{9}}$

$\begin{aligned}\sqrt{\dfrac{5}{9}} &= \dfrac{\sqrt{5}}{\sqrt{9}} && \textit{Quotient Property of Square Roots}\\ &= \dfrac{\sqrt{5}}{3} && \textit{Simplify.}\end{aligned}$

B $\sqrt{\dfrac{a^5}{81a}}$

$\begin{aligned}\sqrt{\dfrac{a^5}{81a}} &= \sqrt{\dfrac{a^4}{81}} && \textit{Simplify.}\\ &= \dfrac{\sqrt{a^4}}{\sqrt{81}} && \textit{Quotient Property of Square Roots}\\ &= \dfrac{a^2}{9} && \textit{Simplify.}\end{aligned}$

Simplify. All variables represent nonnegative numbers.

3a. $\sqrt{\dfrac{12}{27}}$ 3b. $\sqrt{\dfrac{36}{x^4}}$ 3c. $\sqrt{\dfrac{y^6}{4}}$

EXAMPLE 4 **Using the Product and Quotient Properties Together**

Simplify. All variables represent nonnegative numbers.

Caution!

In the expression $\dfrac{4\sqrt{5}}{5}$, $\sqrt{5}$ and 5 are not common factors. $\dfrac{4\sqrt{5}}{5}$ is completely simplified.

A $\sqrt{\dfrac{80}{25}}$

$\dfrac{\sqrt{80}}{\sqrt{25}}$ *Quotient Property*

$\dfrac{\sqrt{16(5)}}{\sqrt{25}}$ *Write 80 as 16(5).*

$\dfrac{\sqrt{16}\,\sqrt{5}}{\sqrt{25}}$ *Product Property*

$\dfrac{4\sqrt{5}}{5}$ *Simplify.*

B $\sqrt{\dfrac{4x^5}{9}}$

$\dfrac{\sqrt{4x^5}}{\sqrt{9}}$ *Quotient Property*

$\dfrac{\sqrt{4}\,\sqrt{x^5}}{\sqrt{9}}$ *Product Property*

$\dfrac{\sqrt{4}\,\sqrt{x^4}\,\sqrt{x}}{\sqrt{9}}$

$\dfrac{2x^2\sqrt{x}}{3}$ *Simplify.*

Simplify. All variables represent nonnegative numbers.

4a. $\sqrt{\dfrac{20}{49}}$ 4b. $\sqrt{\dfrac{z^5}{25y^2}}$ 4c. $\sqrt{\dfrac{p^6}{q^{10}}}$

EXAMPLE 5 **Sports Application**

A baseball diamond is a square with sides of 90 feet. How far is a throw from third base to first base? Give the answer as a radical expression in simplest form. Then estimate the length to the nearest tenth of a foot.

The distance from third base to first base is the hypotenuse of a right triangle. Use the Pythagorean Theorem: $c^2 = a^2 + b^2$.
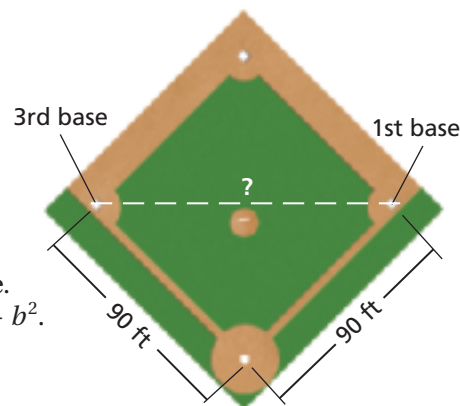
$c = \sqrt{a^2 + b^2}$ *Solve for c.*

$= \sqrt{(90)^2 + (90)^2}$ *Substitute 90 for a and b.*

$= \sqrt{8100 + 8100}$ *Simplify.*

$= \sqrt{16{,}200}$

$= \sqrt{100(81)(2)}$ *Factor 16,200 using perfect squares.*

$= \sqrt{100}\,\sqrt{81}\,\sqrt{2}$ *Use the Product Property of Square Roots.*

$= 10(9)\sqrt{2}$

$= 90\sqrt{2}$ *Simplify.*

≈ 127.3 *Use a calculator and round to the nearest tenth.*

The distance is $90\sqrt{2}$, or about 127.3, feet.

3rd base   1st base   ?   90 ft   90 ft

5. A softball diamond is a square with sides of 60 feet. How long is a throw from third base to first base in softball? Give the answer as a radical expression in simplest form. Then estimate the length to the nearest tenth of a foot.

THINK AND DISCUSS

1. Show two ways to evaluate $\sqrt{16(9)}$.

2. Show two ways to evaluate $\sqrt{\dfrac{100}{4}}$.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write the property and give an example.

	Product Property of Square Roots	Quotient Property of Square Roots
Words		
Example		

11-6 Exercises

GUIDED PRACTICE

1. Vocabulary In the expression $\sqrt{3x - 6} + 7$, what is the *radicand*?

SEE EXAMPLE 1
p. 805

Simplify each expression.

2. $\sqrt{81}$

3. $\sqrt{\dfrac{98}{2}}$

4. $\sqrt{(a + 7)^2}$

SEE EXAMPLE 2
p. 806

Simplify. All variables represent nonnegative numbers.

5. $\sqrt{180}$

6. $\sqrt{40}$

7. $\sqrt{648}$

8. $\sqrt{m^5 n^3}$

9. $\sqrt{32x^4 y^3}$

10. $\sqrt{200a^2 b}$

SEE EXAMPLE 3
p. 806

11. $\sqrt{\dfrac{17}{25}}$

12. $\sqrt{\dfrac{7}{16}}$

13. $\sqrt{\dfrac{6}{49}}$

14. $\sqrt{\dfrac{b}{c^2}}$

15. $\sqrt{\dfrac{4x^2}{36x}}$

16. $\sqrt{\dfrac{7a^4}{9a^3}}$

SEE EXAMPLE 4
p. 807

17. $\sqrt{\dfrac{108}{49}}$

18. $\sqrt{\dfrac{204}{25}}$

19. $\sqrt{\dfrac{512}{81}}$

20. $\sqrt{\dfrac{1}{36x^2}}$

21. $\sqrt{\dfrac{50x^2}{169}}$

22. $\sqrt{\dfrac{72x^7}{4x^4}}$

SEE EXAMPLE 5
p. 807

23. Recreation Your boat is traveling due north from a dock. Your friend's boat left at the same time from the same dock and is headed due east. After an hour, your friend calls and tells you that he has just stopped because of engine trouble. How far must you travel to meet your friend? Give your answer as a radical expression in simplest form. Then estimate the distance to the nearest mile.

PRACTICE AND PROBLEM SOLVING
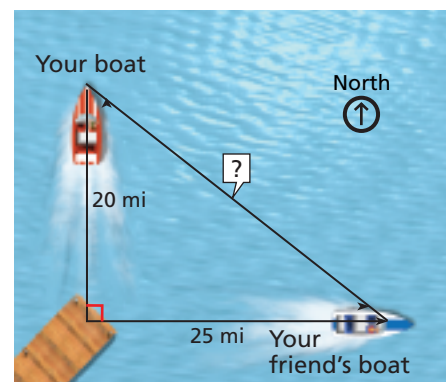
Simplify.

24. $\sqrt{100}$ **25.** $\sqrt{\dfrac{800}{2}}$ **26.** $\sqrt{3^2 + 4^2}$ **27.** $\sqrt{3 \cdot 27}$

28. $\sqrt{a^4}$ **29.** $\sqrt{(x+1)^2}$ **30.** $\sqrt{(5-x)^2}$ **31.** $\sqrt{(x-3)^2}$

Simplify. All variables represent nonnegative numbers.

32. $\sqrt{125}$ **33.** $\sqrt{4000}$ **34.** $\sqrt{216a^2b^2}$ **35.** $\sqrt{320r^2s^2}$

36. $\sqrt{\dfrac{15}{64}}$ **37.** $\sqrt{\dfrac{45}{4}}$ **38.** $\sqrt{\dfrac{64a^4}{4a^6}}$ **39.** $\sqrt{\dfrac{14z^3}{9z^3}}$

40. $\sqrt{\dfrac{128}{81}}$ **41.** $\sqrt{\dfrac{x^3}{y^6}}$ **42.** $\sqrt{\dfrac{150}{196x^2}}$ **43.** $\sqrt{\dfrac{192s^3}{49s}}$

44. Amusement Parks A thrill ride at an amusement park carries riders 160 feet straight up and then releases them for a free fall. The time t in seconds that it takes an object in free fall to reach the ground is $t = \sqrt{\dfrac{d}{16}}$, where d is the distance in feet that it falls. How long does it take the riders to reach the ground? Give your answer as a radical expression in simplest form. Then estimate the answer to the nearest tenth of a second.

Simplify. All variables represent nonnegative numbers.

45. $-4\sqrt{75}$ **46.** $-\sqrt{80}$ **47.** $5x\sqrt{63}$ **48.** $3\sqrt{48x}$

49. $2\sqrt{\dfrac{x^2}{4}}$ **50.** $\dfrac{1}{2}\sqrt{\dfrac{1}{25}}$ **51.** $3x\sqrt{\dfrac{x^5}{81}}$ **52.** $\dfrac{12}{x}\sqrt{\dfrac{x^2y}{36}}$

Use the Product Property or the Quotient Property of Square Roots to write each expression as a single square root. Then simplify if possible.

53. $\sqrt{12}\,\sqrt{3}$ **54.** $\sqrt{18}\,\sqrt{8}$ **55.** $\sqrt{10}\,\sqrt{5}$ **56.** $\sqrt{8}\,\sqrt{14}$

57. $\dfrac{\sqrt{33}}{\sqrt{11}}$ **58.** $\dfrac{\sqrt{24}}{\sqrt{2}}$ **59.** $\dfrac{\sqrt{60}}{\sqrt{3}}$ **60.** $\dfrac{\sqrt{72}}{\sqrt{9}}$

61. Multi-Step How many whole feet of fencing would be needed to enclose the triangular garden that is sketched at right? Explain your answer.
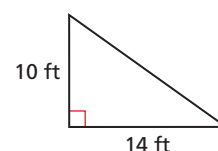
62. Write About It Write a series of steps that you could use to simplify $\sqrt{\dfrac{28}{49}}$.

10 ft   14 ft

63. This problem will prepare you for the Multi-Step Test Prep on page 830.

  a. The velocity of a roller coaster in feet per second at the bottom of a hill is $v = \sqrt{64h}$, where h is the hill's height in feet. Simplify this expression. Then estimate the velocity at the bottom of a 137-foot hill.

  b. The distance along the track of a hill is $d = \sqrt{x^2 + h^2}$, where x is the horizontal distance along the ground and h is the hill's height. Where does this equation come from?

  c. For the hill in part **a,** the horizontal distance along the ground is 103 feet. What is the distance along the track? Round your answer to the nearest tenth.

64. Critical Thinking The Product Property of Square Roots states that $\sqrt{ab} = \sqrt{a}\sqrt{b}$, where $a \geq 0$ and $b \geq 0$. Why must a and b be greater than or equal to zero?

65. Architecture The formula $d = \frac{\sqrt{6h}}{3}$ estimates the distance d in miles that a person can see to the horizon from h feet above the ground. Find the distance you could see to the horizon from the top of each building in the graph. Give your answers as radical expressions in simplest form and as estimates to the nearest tenth of a mile.

Tall Buildings

Height (ft): 0, 300, 600, 900, 1200, 1500

- Sears Tower: 1450
- Empire State Building: 1250
- Aon Tower: 1136

Building
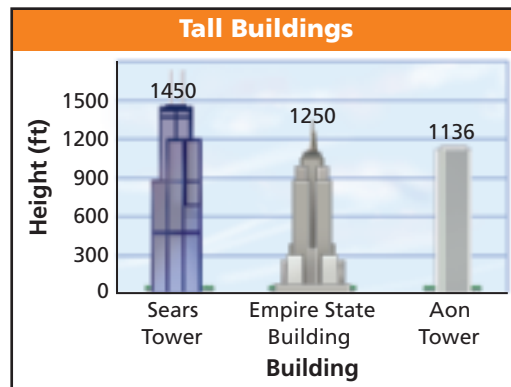
66. Math History Heron's formula for the area A of a triangle is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the side lengths and $s = \frac{1}{2}(a+b+c)$. Find the area of a triangle with side lengths of 7 m, 9 m, and 12 m. Give your answer as a radical expression in simplest form and as an estimate to the nearest tenth.

TEST PREP

67. Which expression is in simplest form?
Ⓐ $\sqrt{49}$ Ⓑ $\sqrt{48}$ Ⓒ $\sqrt{35}$ Ⓓ $\sqrt{36}$

68. Which expression is equal to $\sqrt{60}$?
Ⓕ $2\sqrt{15}$ Ⓖ $6\sqrt{10}$ Ⓗ $15\sqrt{2}$ Ⓙ $10\sqrt{60}$

69. How long is the diagonal of a square whose area is 100 square feet?
Ⓐ $2\sqrt{10}$ feet Ⓑ 10 feet Ⓒ $10\sqrt{2}$ feet Ⓓ 20 feet

CHALLENGE AND EXTEND

Simplify. All variables represent nonnegative numbers.

70. $\sqrt{4x+16}$ **71.** $\sqrt{x^3 + x^2}$ **72.** $\sqrt{9x^3 - 18x^2}$

73. Let x represent any real number (including negative numbers). Simplify each of the following expressions, using absolute-value symbols when necessary.
  a. $\sqrt{x^2}$ **b.** $\sqrt{x^4}$ **c.** $\sqrt{x^6}$ **d.** $\sqrt{x^8}$ **e.** $\sqrt{x^{10}}$
  f. For any nonnegative integer n, $\sqrt{x^{2n}} = \blacksquare$ if n is even, and $\sqrt{x^{2n}} = \blacksquare$ if n is odd.

SPIRAL REVIEW

Tell whether each relationship is a direct variation. Explain. *(Lesson 5-5)*

74.

x	2	3	4
y	12	18	24

75.

x	2	3	4
y	−6	−5	−4

76. Write an equation in slope-intercept form for the line through $(3, 1)$ and $(2, -5)$. *(Lesson 5-6)*

Graph each data set. Which kind of model best describes the data? *(Lesson 11-4)*

77. $\{(-3, 16), (-2, 8), (0, 2), (1, 1), (3, 0.25)\}$
78. $\{(-5, 15), (-2, -6), (0, -10), (3, -1), (4, 6)\}$

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11-7 Adding and Subtracting Radical Expressions

Objective
Add and subtract radical expressions.

Vocabulary
like radicals

Why learn this?
You can add or subtract radical expressions to find perimeter. (See Example 3.)

Square-root expressions with the same radicand are examples of **like radicals**.

Like Radicals	$2\sqrt{5}$ and $4\sqrt{5}$	$6\sqrt{x}$ and $-2\sqrt{x}$	$3\sqrt{4t}$ and $\sqrt{4t}$
Unlike Radicals	2 and $\sqrt{15}$	$6\sqrt{x}$ and $\sqrt{6x}$	$3\sqrt{2}$ and $2\sqrt{3}$

Like radicals can be combined by adding or subtracting. You can use the Distributive Property to show how this is done:

$$2\sqrt{5} + 4\sqrt{5} = (2 + 4)\sqrt{5} = 6\sqrt{5}$$

$$6\sqrt{x} - 2\sqrt{x} = (6 - 2)\sqrt{x} = 4\sqrt{x}$$

Notice that you can combine like radicals by adding or subtracting the numbers multiplied by the radical and keeping the radical the same.

EXAMPLE 1 Adding and Subtracting Square-Root Expressions

Add or subtract.

A $3\sqrt{7} + 8\sqrt{7}$
$3\sqrt{7} + 8\sqrt{7}$ *The terms are like radicals.*
$11\sqrt{7}$

B $9\sqrt{y} - \sqrt{y}$
$9\sqrt{y} - 1\sqrt{y}$ *$\sqrt{y} = 1\sqrt{y}$; the terms are like radicals.*
$8\sqrt{y}$

C $12\sqrt{2} - 4\sqrt{11}$
$12\sqrt{2} - 4\sqrt{11}$ *The terms are unlike radicals. Do not combine.*

D $-8\sqrt{3d} + 6\sqrt{2d} + 10\sqrt{3d}$
$-8\sqrt{3d} + 6\sqrt{2d} + 10\sqrt{3d}$ *Identify like radicals.*
$2\sqrt{3d} + 6\sqrt{2d}$ *Combine like radicals.*

Helpful Hint
Combining like radicals is similar to combining like terms.
$2\sqrt{5} + 4\sqrt{5} = 6\sqrt{5}$
$2x + 4x = 6x$

CHECK IT OUT!
Add or subtract.
1a. $5\sqrt{7} - 6\sqrt{7}$ **1b.** $8\sqrt{3} - 5\sqrt{3}$
1c. $4\sqrt{n} + 4\sqrt{n}$ **1d.** $\sqrt{2s} - \sqrt{5s} + 9\sqrt{5s}$

Sometimes radicals do not appear to be like until they are simplified. Simplify all radicals in an expression before trying to identify like radicals.

EXAMPLE 2 **Simplifying Before Adding or Subtracting**

Simplify each expression.

A $\sqrt{12} + \sqrt{27}$

$\sqrt{4(3)} + \sqrt{9(3)}$ *Factor the radicands using perfect squares.*

$\sqrt{4}\sqrt{3} + \sqrt{9}\sqrt{3}$ *Product Property of Square Roots*

$2\sqrt{3} + 3\sqrt{3}$ *Simplify.*

$5\sqrt{3}$ *Combine like radicals.*

B $3\sqrt{8} + \sqrt{45}$

$3\sqrt{4(2)} + \sqrt{9(5)}$ *Factor the radicands using perfect squares.*

$3\sqrt{4}\sqrt{2} + \sqrt{9}\sqrt{5}$ *Product Property of Square Roots*

$3(2)\sqrt{2} + 3\sqrt{5}$ *Simplify.*

$6\sqrt{2} + 3\sqrt{5}$ *The terms are unlike radicals. Do not combine.*

> **Remember!**
>
> When you write a radicand as a product, make at least one factor a perfect square.

C $5\sqrt{28x} - 8\sqrt{7x}$

$5\sqrt{4(7x)} - 8\sqrt{7x}$ *Factor 28x using a perfect square.*

$5\sqrt{4}\sqrt{7x} - 8\sqrt{7x}$ *Product Property of Square Roots*

$5(2)\sqrt{7x} - 8\sqrt{7x}$ *Simplify.*

$10\sqrt{7x} - 8\sqrt{7x}$

$2\sqrt{7x}$ *Combine like radicals.*

D $\sqrt{125b} + 3\sqrt{20b} - \sqrt{45b}$

$\sqrt{25(5b)} + 3\sqrt{4(5b)} - \sqrt{9(5b)}$ *Factor the radicands using perfect squares.*

$\sqrt{25}\sqrt{5b} + 3\sqrt{4}\sqrt{5b} - \sqrt{9}\sqrt{5b}$ *Product Property of Square Roots*

$5\sqrt{5b} + 3(2)\sqrt{5b} - 3\sqrt{5b}$ *Simplify.*

$5\sqrt{5b} + 6\sqrt{5b} - 3\sqrt{5b}$

$8\sqrt{5b}$ *Combine like radicals.*

CHECK IT OUT! Simplify each expression.

2a. $\sqrt{54} + \sqrt{24}$ **2b.** $4\sqrt{27} - \sqrt{18}$ **2c.** $\sqrt{12y} + \sqrt{27y}$

EXAMPLE 3 *Geometry Application*

Find the perimeter of the triangle. Give your answer as a radical expression in simplest form.

$5\sqrt{7}$ cm

$\sqrt{28}$ cm

12 cm
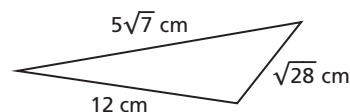
$12 + 5\sqrt{7} + \sqrt{28}$ *Write an expression for perimeter.*

$12 + 5\sqrt{7} + \sqrt{4(7)}$ *Factor 28 using a perfect square.*

$12 + 5\sqrt{7} + \sqrt{4}\sqrt{7}$ *Product Property of Square Roots*

$12 + 5\sqrt{7} + 2\sqrt{7}$ *Simplify.*

$12 + 7\sqrt{7}$ *Combine like radicals.*

The perimeter is $\left(12 + 7\sqrt{7}\right)$ cm.

CHECK IT OUT! **3.** Find the perimeter of a rectangle whose length is $2\sqrt{b}$ inches and whose width is $3\sqrt{b}$ inches. Give your answer as a radical expression in simplest form.
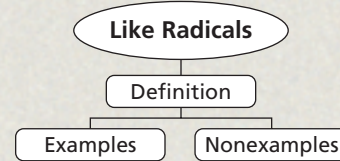
11-7 Exercises
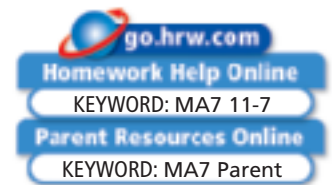
GUIDED PRACTICE

1. Vocabulary Give an example of *like radicals*.

SEE EXAMPLE 1
p. 811

Add or subtract.

2. $14\sqrt{3} - 6\sqrt{3}$

3. $9\sqrt{5} + \sqrt{5}$

4. $6\sqrt{2} + 5\sqrt{2} - 15\sqrt{2}$

5. $3\sqrt{7} + 5\sqrt{2}$

6. $5\sqrt{a} - 9\sqrt{a}$

7. $9\sqrt{6a} + 6\sqrt{5a} - 4\sqrt{6a}$

SEE EXAMPLE 2
p. 812

Simplify each expression.

8. $\sqrt{32} - \sqrt{8}$

9. $4\sqrt{12} + \sqrt{75}$

10. $2\sqrt{3} + 5\sqrt{12} - \sqrt{27}$

11. $\sqrt{20x} - \sqrt{45x}$

12. $\sqrt{28c} + 9\sqrt{24c}$

13. $\sqrt{50t} - 2\sqrt{12t} + 3\sqrt{2t}$

SEE EXAMPLE 3
p. 812

14. Geometry Find the perimeter of the trapezoid shown. Give your answer as a radical expression in simplest form.

$\sqrt{50}$ in.
$\sqrt{8}$ in.
$\sqrt{8}$ in.
$\sqrt{18}$ in.
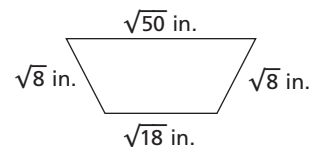
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–20	1
21–29	2
30	3

Add or subtract.

15. $4\sqrt{3} + 2\sqrt{3}$

16. $\frac{1}{2}\sqrt{72} - 12$

17. $2\sqrt{11} + \sqrt{11} - 6\sqrt{11}$

18. $6\sqrt{7} + 7\sqrt{6}$

19. $-3\sqrt{n} - \sqrt{n}$

20. $2\sqrt{2y} + 3\sqrt{2y} - 2\sqrt{3y}$

Extra Practice
Skills Practice p. S25
Application Practice p. S38

Simplify each expression.

21. $\sqrt{175} + \sqrt{28}$

22. $2\sqrt{80} - \sqrt{20}$

23. $5\sqrt{8} - \sqrt{32} + 2\sqrt{18}$

24. $\sqrt{150r} + \sqrt{54r}$

25. $\sqrt{63x} - 4\sqrt{27x}$

26. $\sqrt{48p} + 3\sqrt{18p} - 2\sqrt{27p}$

27. $\sqrt{180j} - \sqrt{45j}$

28. $3\sqrt{90c} - \sqrt{40c}$

29. $2\sqrt{75m} - \sqrt{12m} - \sqrt{108m}$

30. Fitness What is the total length of the jogging path? Give your answer as a radical expression in simplest form.
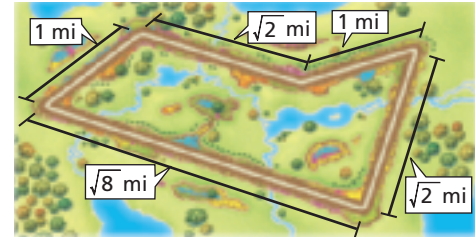
Simplify each expression.

31. $5\sqrt{7} + 7\sqrt{7}$　　**32.** $18\sqrt{ab} - 10\sqrt{ab}$

33. $-3\sqrt{3} + 3\sqrt{3}$　　**34.** $\sqrt{98} + \sqrt{128}$

35. $\sqrt{300} - \sqrt{27}$　　**36.** $\sqrt{45x} + \sqrt{500x}$

37. $\dfrac{5}{2}\sqrt{8} + \dfrac{\sqrt{32}}{2}$　　**38.** $\dfrac{1}{6}\sqrt{18} - \dfrac{\sqrt{2}}{2}$

39. Geometry Use the diagram to answer the following:

　a. What is the area of section A? section B? section C?

　b. What is the combined area of the three sections?

　c. Explain how this model relates to adding like radicals.
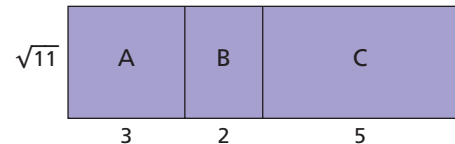
Simplify each expression.

40. $\sqrt{450ab} - \sqrt{50ab}$　　　　**41.** $\sqrt{12} + \sqrt{125} + \sqrt{25}$

42. $\sqrt{338} - \sqrt{18}$　　　　**43.** $\sqrt{700x} - \sqrt{28x} - \sqrt{70x}$

44. $-3\sqrt{90} - 3\sqrt{160}$　　　　**45.** $7\sqrt{80k} + 2\sqrt{20k} + \sqrt{45k}$

46. $\sqrt{24abc} + \sqrt{600abc}$　　　　**47.** $\sqrt{12} + \sqrt{20} + \sqrt{27} + \sqrt{45}$

48. /// **ERROR ANALYSIS** /// Which expressions are simplified incorrectly? Explain the error in each incorrect simplification.

A

$4\sqrt{5} + 5\sqrt{5}$

$9\sqrt{10}$

B

$8\sqrt{3} - 7\sqrt{3}$

$\sqrt{3}$

C

$7\sqrt{11} - 3\sqrt{6}$

$4\sqrt{5}$

49. Write About It Tell how to identify like radicals. Give examples and nonexamples of like radicals in your answer.

Complete each box to make a true statement.

50. $5\sqrt{ab} + 2\sqrt{\boxed{}} - 3\sqrt{a} = 7\sqrt{ab} - 3\sqrt{a}$　　**51.** $4\sqrt{x} - \sqrt{\boxed{}x} = \sqrt{x}$

52. $5\sqrt{2} - \sqrt{\boxed{}} + \sqrt{2} = 4\sqrt{2}$　　**53.** $\sqrt{\boxed{}} + 8\sqrt{2} = 11\sqrt{2}$

54. $3\sqrt{3} + 2\sqrt{3} + \sqrt{\boxed{}} = 9\sqrt{3}$　　**55.** $2x - \sqrt{\boxed{}} = -4x$

MULTI-STEP TEST PREP

56. This problem will prepare you for the Multi-Step Test Prep on page 830.

　a. The first Ferris wheel was designed by George W. Ferris and introduced at the 1893 Chicago World's Fair. Its diameter was 250 feet. What was its radius?

　b. For a rider halfway up on the ride, the distance from the boarding point can be found by using the equation $d = \sqrt{2r^2}$, where r is the radius of the wheel. Explain where this equation comes from. (*Hint:* Draw a picture.)

57. Multi-Step A square has an area of 48 in². Another square has an area of 12 in². Write a simplified radical expression for the perimeter of each square. Then write a simplified radical expression for the combined perimeters of the two squares.

58. Critical Thinking How are like radicals similar to like terms?

59. Which of the following expressions CANNOT be simplified?

Ⓐ $3\sqrt{5} + 4\sqrt{5}$ Ⓒ $2\sqrt{8} + 3\sqrt{2}$

Ⓑ $5\sqrt{6} + 6\sqrt{5}$ Ⓓ $3\sqrt{12} + \sqrt{27}$

60. What is $-5\sqrt{7x} + 6\sqrt{7x}$?

Ⓕ $\sqrt{7x}$ Ⓖ $\sqrt{14x^2}$ Ⓗ $\sqrt{14x}$ Ⓙ $7x$

61. What is $\sqrt{18} - \sqrt{2}$?

Ⓐ $2\sqrt{2}$ Ⓑ 4 Ⓒ $4\sqrt{2}$ Ⓓ $8\sqrt{2}$

CHALLENGE AND EXTEND

Simplify. All variables represent nonnegative numbers.

62. $5\sqrt{x-5} + 2\sqrt{x-5}$ **63.** $x\sqrt{x} + 2\sqrt{x}$

64. $4\sqrt{x-3} + \sqrt{25x-75}$ **65.** $2\sqrt{x+7} - \sqrt{4x+28}$

66. $\sqrt{4x^3 + 24x^2} + \sqrt{x^3 + 6x^2}$ **67.** $\sqrt{x^3 - x^2} + \sqrt{4x - 4}$

68. $\sqrt{x^3 + 2x^2} - \sqrt{x+2}$ **69.** $\sqrt{9x+9} - \sqrt{x^3 + 2x^2}$

70. Geometry Find the area of the trapezoid. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$.

$\sqrt{20}$ cm $\sqrt{45}$ cm $4\sqrt{5}$ cm
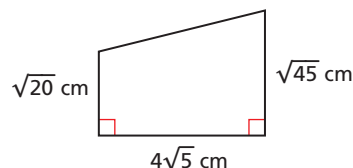
SPIRAL REVIEW

71. Use slope to show that $ABCD$ is a parallelogram. *(Lesson 5-8)*

72. Use slope to show that XYZ is a right triangle. *(Lesson 5-8)*
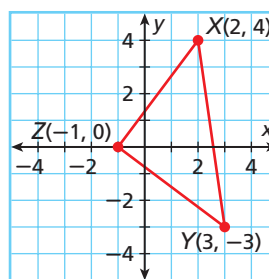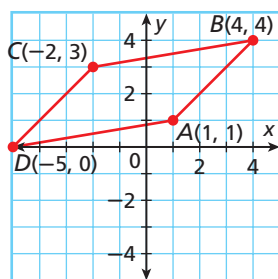
73. A number cube is rolled and a coin is tossed. What is the probability of rolling a 6 and tossing heads? *(Lesson 10-7)*

Find the domain of each square-root function. *(Lesson 11-5)*

74. $y = \sqrt{4x-2}$ **75.** $y = -2\sqrt{x+3}$ **76.** $y = 1 + \sqrt{x+6}$

11-8 Multiplying and Dividing Radical Expressions

Objectives
Multiply and divide radical expressions.

Rationalize denominators.

Who uses this?
Electricians can divide radical expressions to find how much current runs through an appliance. (See Exercise 64.)

You can use the Product and Quotient Properties of square roots you have already learned to multiply and divide expressions containing square roots.

DON'T GO IN THERE!
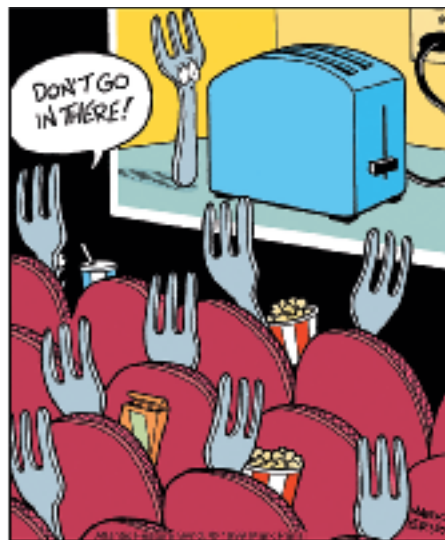
Off the Mark. Cartoon copyrighted by Mark Parisi, printed with permission.

EXAMPLE 1 **Multiplying Square Roots**

Multiply. Write each product in simplest form.

A $\sqrt{3}\sqrt{6}$

$\sqrt{3(6)}$	*Product Property of Square Roots*
$\sqrt{18}$	*Multiply the factors in the radicand.*
$\sqrt{9(2)}$	*Factor 18 using a perfect-square factor.*
$\sqrt{9}\sqrt{2}$	*Product Property of Square Roots*
$3\sqrt{2}$	*Simplify.*

B $\left(5\sqrt{3}\right)^2$

$5\sqrt{3}\cdot 5\sqrt{3}$	*Expand the expression.*
$5(5)\sqrt{3}\sqrt{3}$	*Commutative Property of Multiplication*
$25\sqrt{3(3)}$	*Product Property of Square Roots*
$25\sqrt{9}$	*Simplify the radicand.*
$25(3)$	*Simplify the square root.*
75	*Multiply.*

C $2\sqrt{8x}\sqrt{4x}$

$2\sqrt{8x(4x)}$	*Product Property of Square Roots*
$2\sqrt{32x^2}$	*Multiply the factors in the radicand.*
$2\sqrt{16(2)x^2}$	*Factor 32 using a perfect-square factor.*
$2\sqrt{16}\sqrt{2}\sqrt{x^2}$	*Product Property of Square Roots*
$2(4)\sqrt{2}(x)$	
$8x\sqrt{2}$	

Helpful Hint

In Example 1C, $\sqrt{8x}$ and $\sqrt{4x}$ represent real numbers only if $x \geq 0$, so, in this case, $\sqrt{x^2} = x$.

CHECK IT OUT!

Multiply. Write each product in simplest form.

1a. $\sqrt{5}\sqrt{10}$ **1b.** $\left(3\sqrt{7}\right)^2$ **1c.** $\sqrt{2m}\sqrt{14m}$

EXAMPLE 2

Using the Distributive Property

Multiply. Write each product in simplest form.

A $\sqrt{2}\left(5 + \sqrt{12}\right)$

$\sqrt{2}\left(5 + \sqrt{12}\right)$

$\sqrt{2}(5) + \sqrt{2}\sqrt{12}$ *Distribute $\sqrt{2}$.*

$5\sqrt{2} + \sqrt{2(12)}$ *Product Property of Square Roots*

$5\sqrt{2} + \sqrt{24}$ *Multiply the factors in the second radicand.*

$5\sqrt{2} + \sqrt{4(6)}$ *Factor 24 using a perfect-square factor.*

$5\sqrt{2} + \sqrt{4}\sqrt{6}$ *Product Property of Square Roots*

$5\sqrt{2} + 2\sqrt{6}$ *Simplify.*

B $\sqrt{3}\left(\sqrt{3} - \sqrt{5}\right)$

$\sqrt{3}\left(\sqrt{3} - \sqrt{5}\right)$

$\sqrt{3}\sqrt{3} - \sqrt{3}\sqrt{5}$ *Distribute $\sqrt{3}$.*

$\sqrt{3(3)} - \sqrt{3(5)}$ *Product Property of Square Roots*

$\sqrt{9} - \sqrt{15}$ *Simplify the radicands.*

$3 - \sqrt{15}$ *Simplify.*

CHECK IT OUT! **Multiply. Write each product in simplest form.**

2a. $\sqrt{6}\left(\sqrt{8} - 3\right)$ **2b.** $\sqrt{5}\left(\sqrt{10} + 4\sqrt{3}\right)$

2c. $\sqrt{7k}\left(\sqrt{7} - 5\right)$ **2d.** $5\sqrt{5}\left(-4 + 6\sqrt{5}\right)$

Remember!

First terms
Outer terms
Inner terms
Last terms

See Lesson 7-7.

In Chapter 7, you learned to multiply binomials by using the FOIL method. The same method can be used to multiply square-root expressions that contain two terms.

$$\left(4 + \sqrt{3}\right)\left(5 + \sqrt{3}\right) = 4(5) + 4\sqrt{3} + 5\sqrt{3} + \sqrt{3}\sqrt{3}$$

$$= 20 \qquad + 9\sqrt{3} \qquad + 3 \quad = \quad 23 + 9\sqrt{3}$$

EXAMPLE 3

Multiplying Sums and Differences of Radicals

Multiply. Write each product in simplest form.

A $\left(4 + \sqrt{5}\right)\left(3 - \sqrt{5}\right)$

$12 - 4\sqrt{5} + 3\sqrt{5} - 5$ *Use the FOIL method.*

$7 - \sqrt{5}$ *Simplify by combining like terms.*

B $\left(\sqrt{7} - 5\right)^{2}$

$\left(\sqrt{7} - 5\right)\left(\sqrt{7} - 5\right)$ *Expand the expression.*

$7 - 5\sqrt{7} - 5\sqrt{7} + 25$ *Use the FOIL method.*

$32 - 10\sqrt{7}$ *Simplify by combining like terms.*

CHECK IT OUT! **Multiply. Write each product in simplest form.**

3a. $\left(3 + \sqrt{3}\right)\left(8 - \sqrt{3}\right)$ **3b.** $\left(9 + \sqrt{2}\right)^{2}$

3c. $\left(3 - \sqrt{2}\right)^{2}$ **3d.** $\left(4 - \sqrt{3}\right)\left(\sqrt{3} + 5\right)$

A quotient with a square root in the denominator is **not** simplified. To simplify these expressions, multiply by a form of 1 to get a perfect-square radicand in the denominator. This is called *rationalizing the denominator*.

EXAMPLE **4** **Rationalizing the Denominator**

Simplify each quotient.

A $\dfrac{\sqrt{7}}{\sqrt{2}}$

$\dfrac{\sqrt{7}}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right)$ *Multiply by a form of 1 to get a perfect-square radicand in the denominator.*

$\dfrac{\sqrt{14}}{\sqrt{4}}$ *Product Property of Square Roots*

$\dfrac{\sqrt{14}}{2}$ *Simplify the denominator.*

> **Helpful Hint**
>
> Use the square root in the denominator to find the appropriate form of 1 for multiplication.

B $\dfrac{\sqrt{7}}{\sqrt{8n}}$

$\dfrac{\sqrt{7}}{\sqrt{4(2n)}}$ *Write 8n using a perfect-square factor.*

$\dfrac{\sqrt{7}}{2\sqrt{2n}}$ *Simplify the denominator.*

$\dfrac{\sqrt{7}}{2\sqrt{2n}}\left(\dfrac{\sqrt{2n}}{\sqrt{2n}}\right)$ *Multiply by a form of 1 to get a perfect-square radicand in the denominator.*

$\dfrac{\sqrt{14n}}{2\sqrt{4n^2}}$ *Product Property of Square Roots*

$\dfrac{\sqrt{14n}}{2(2n)}$ *Simplify the square root in the denominator.*

$\dfrac{\sqrt{14n}}{4n}$ *Simplify the denominator.*

CHECK IT OUT! Simplify each quotient.

4a. $\dfrac{\sqrt{13}}{\sqrt{5}}$ **4b.** $\dfrac{\sqrt{7a}}{\sqrt{12}}$ **4c.** $\dfrac{2\sqrt{80}}{\sqrt{7}}$

THINK AND DISCUSS

1. Explain why multiplying $\dfrac{\sqrt{6}}{\sqrt{5}}$ by $\dfrac{\sqrt{5}}{\sqrt{5}}$ does not change the value of $\dfrac{\sqrt{6}}{\sqrt{5}}$.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give an example and show how to simplify it.

Know it! Note

Multiplying Radical Expressions
- Multiplying two square roots
- Using the Distributive Property
- Using FOIL

11-8

Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 11-8
Parent Resources Online
KEYWORD: MA7 Parent
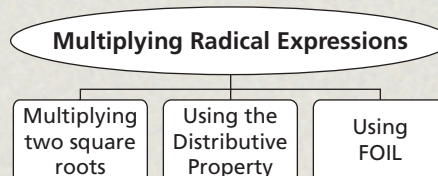
GUIDED PRACTICE

SEE EXAMPLE 1
p. 816

Multiply. Write each product in simplest form.

1. $\sqrt{2}\sqrt{3}$

2. $\sqrt{3}\sqrt{8}$

3. $\left(5\sqrt{5}\right)^2$

4. $\left(4\sqrt{2}\right)^2$

5. $3\sqrt{3a}\sqrt{10}$

6. $2\sqrt{15p}\sqrt{3p}$

SEE EXAMPLE 2
p. 817

7. $\sqrt{6}\left(2+\sqrt{7}\right)$

8. $\sqrt{3}\left(5-\sqrt{3}\right)$

9. $\sqrt{7}\left(\sqrt{5}-\sqrt{3}\right)$

10. $\sqrt{2}\left(\sqrt{10}+8\sqrt{2}\right)$

11. $\sqrt{5y}\left(\sqrt{15}+4\right)$

12. $\sqrt{2t}\left(\sqrt{6t}-\sqrt{2t}\right)$

SEE EXAMPLE 3
p. 817

13. $\left(2+\sqrt{2}\right)\left(5+\sqrt{2}\right)$

14. $\left(4+\sqrt{6}\right)\left(3-\sqrt{6}\right)$

15. $\left(\sqrt{3}-4\right)\left(\sqrt{3}+2\right)$

16. $\left(5+\sqrt{3}\right)^2$

17. $\left(\sqrt{6}-5\sqrt{3}\right)^2$

18. $\left(6+3\sqrt{2}\right)^2$

SEE EXAMPLE 4
p. 818

Simplify each quotient.

19. $\dfrac{\sqrt{13}}{\sqrt{2}}$

20. $\dfrac{\sqrt{20}}{\sqrt{8}}$

21. $\dfrac{\sqrt{11}}{6\sqrt{3}}$

22. $\dfrac{\sqrt{28}}{\sqrt{3s}}$

23. $\dfrac{2}{\sqrt{7}}$

24. $\dfrac{3}{\sqrt{6}}$

25. $\dfrac{1}{\sqrt{5x}}$

26. $\dfrac{\sqrt{3}}{\sqrt{x}}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
27–32	1
33–38	2
39–44	3
45–52	4

Extra Practice
Skills Practice p. S25
Application Practice p. S38

Multiply. Write each product in simplest form.

27. $\sqrt{3}\sqrt{5}\sqrt{6}$

28. $3\sqrt{6}\left(5\sqrt{6}\right)$

29. $\left(2\sqrt{2}\right)^2$

30. $\left(3\sqrt{6}\right)^2$

31. $\sqrt{21d}\left(2\sqrt{3d}\right)$

32. $4\sqrt{5n}\left(2\sqrt{5n}\right)\left(3\sqrt{3n}\right)$

33. $\sqrt{5}\left(4-\sqrt{10}\right)$

34. $\sqrt{2}\left(\sqrt{6}+2\right)$

35. $\sqrt{2}\left(\sqrt{6}-\sqrt{10}\right)$

36. $3\sqrt{3}\left(\sqrt{8}-2\sqrt{6}\right)$

37. $\sqrt{3f}\left(\sqrt{3}+12\right)$

38. $\sqrt{8m}\left(\sqrt{10}+\sqrt{2m}\right)$

39. $\left(15+\sqrt{15}\right)\left(4+\sqrt{15}\right)$

40. $\left(\sqrt{6}+4\right)\left(\sqrt{2}-7\right)$

41. $\left(3-\sqrt{2}\right)\left(4+\sqrt{2}\right)$

42. $\left(\sqrt{5}-5\right)^2$

43. $\left(\sqrt{3}+8\right)^2$

44. $\left(2\sqrt{3}+4\sqrt{5}\right)^2$

Simplify each quotient.

45. $\dfrac{\sqrt{75}}{\sqrt{2}}$

46. $\dfrac{\sqrt{5}}{4\sqrt{8}}$

47. $\dfrac{\sqrt{27}}{3\sqrt{x}}$

48. $\dfrac{\sqrt{48k}}{\sqrt{5}}$

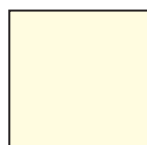49. $\dfrac{\sqrt{49x}}{\sqrt{2}}$

50. $\dfrac{3\sqrt{27}}{\sqrt{b}}$

51. $\dfrac{\sqrt{12y}}{\sqrt{3}}$

52. $\dfrac{\sqrt{12t}}{\sqrt{6}}$

Geometry Find the area of each figure. Give your answer as a radical expression in simplest form.

53.

$6\sqrt{5}$ in.

$6\sqrt{5}$ in.

54.

$\sqrt{6}$ m

$2\sqrt{3}$ m

55.
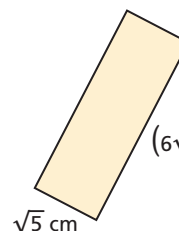
$\left(6\sqrt{2}-2\right)$ cm

$\sqrt{5}$ cm

Simplify.

56. $\sqrt{3}\left(\dfrac{\sqrt{2}}{\sqrt{7}}\right)$

57. $\dfrac{15\sqrt{10}}{5\sqrt{3}}$

58. $\dfrac{6+\sqrt{18}}{3}$

59. $\left(\sqrt{3}-4\right)\left(\sqrt{3}+2\right)$

60. $\sqrt{2}\left(6+\sqrt{12}\right)$

61. $\dfrac{\sqrt{1}+\sqrt{25}}{\sqrt{2}}$

62. $\dfrac{\sqrt{15}+\sqrt{10}}{\sqrt{5}}$

63. $\sqrt{12}\left(\sqrt{3}+8\right)^{2}$

64. $\sqrt{3}\left(4-2\sqrt{5}\right)$

65. $\left(\sqrt{x}-\sqrt{y}\right)^{2}$

66. $\left(\sqrt{x}-5\right)\left(3\sqrt{x}+7\right)$

67. $\left(\sqrt{3}+\sqrt{x}\right)^{2}$

68. Electricity Electrical current in amps can be represented by $\dfrac{\sqrt{W}}{\sqrt{R}}$, where W is power in watts and R is resistance in ohms. How much electrical current is running through a microwave oven that has 850 watts of power and 5 ohms of resistance? Give the answer as a radical expression in simplest form. Then estimate the amount of current to the nearest tenth.

69. Physics The *period* of a pendulum is the amount of time it takes the pendulum to make one complete swing and return to its starting point. The period of a pendulum in seconds can be represented by $2\pi\sqrt{\dfrac{\ell}{32}}$, where ℓ is the length of the pendulum in feet. What is the period of a pendulum whose length is 3 feet? Give the answer as a radical expression in simplest form. Then estimate the period to the nearest tenth.
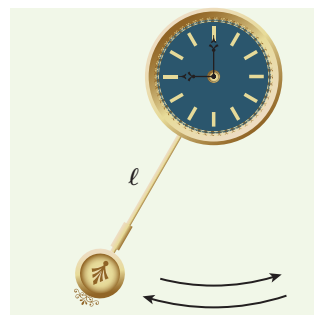
Geometry Find the area of each triangle. Give the exact answer in simplest form. (*Hint:* The formula for the area of a triangle is $A=\frac{1}{2}bh$.)

70.

$\sqrt{3}$ yd

$2\sqrt{6}$ yd

71.

$7\sqrt{11}$ ft

$7\sqrt{11}$ ft

72.

$\left(2\sqrt{3}-3\right)$ cm

$\left(2-\sqrt{3}\right)$ cm
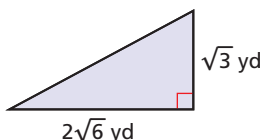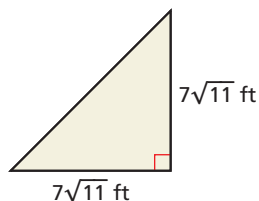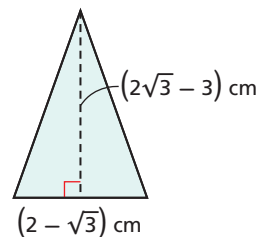
73. Write About It Describe an expression for which you would have to rationalize the denominator. How would you do it? Include an explanation of how you would choose what to multiply the original expression by.

MULTI-STEP TEST PREP

74. This problem will prepare you for the Multi-Step Test Prep on page 830.

  a. Many amusement parks have free-fall rides in which cars travel straight up a tower and then are allowed to fall back to the ground. The time in seconds for any object in free fall is $t=\sqrt{\dfrac{d}{16}}$, where d is the distance that the object falls in feet. On a particular free-fall ride, the cars are in free fall for 100 feet. How long does free fall last on this ride?

  b. The cars in the ride from part **a** travel up the tower at a speed of 18 feet per second. How long does this trip take? Round your answer to the nearest tenth. How does this time compare with the time spent in free fall?

75. What is the product of $3\sqrt{5}$ and $\sqrt{15}$?

   Ⓐ $5\sqrt{3}$ Ⓑ $15\sqrt{3}$ Ⓒ $15\sqrt{15}$ Ⓓ $45\sqrt{5}$

76. Which of the following is the result of rationalizing the denominator in the expression $\frac{4}{3\sqrt{2}}$?

   Ⓕ $\frac{\sqrt{2}}{3}$ Ⓖ $2\sqrt{2}$ Ⓗ $\frac{2\sqrt{2}}{3}$ Ⓙ $\frac{3\sqrt{2}}{2}$

77. Which of the following is equivalent to $\left(5\sqrt{10}\right)^2$?

   Ⓐ 50   Ⓑ 100   Ⓒ 125   Ⓓ 250

CHALLENGE AND EXTEND

The expressions $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are called *conjugates*. Using the FOIL method, conjugates can be multiplied as follows:

$$\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = a + \sqrt{ab} - \sqrt{ab} - b = a - b$$

$$\left(\sqrt{3} - \sqrt{5}\right)\left(\sqrt{3} + \sqrt{5}\right) = 3 + \sqrt{15} - \sqrt{15} - 5 = 3 - 5 = -2$$

Notice that the product does not contain any square roots. This means that you can use conjugates to rationalize denominators that contain sums or differences of square roots:

$$\frac{\sqrt{2}}{\sqrt{7} + \sqrt{2}}\left(\frac{\sqrt{7} - \sqrt{2}}{\sqrt{7} - \sqrt{2}}\right) = \frac{\sqrt{2}\left(\sqrt{7} - \sqrt{2}\right)}{\left(\sqrt{7} + \sqrt{2}\right)\left(\sqrt{7} - \sqrt{2}\right)} = \frac{\sqrt{14} - \sqrt{4}}{7 - 2} = \frac{\sqrt{14} - 2}{5}$$

Simplify.

78. $\dfrac{4}{\sqrt{3} - \sqrt{2}}$ **79.** $\dfrac{8}{\sqrt{3} + \sqrt{5}}$ **80.** $\dfrac{\sqrt{5}}{\sqrt{10} + \sqrt{3}}$ **81.** $\dfrac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$

82. $\dfrac{\sqrt{3}}{\sqrt{2} + \sqrt{3}}$ **83.** $\dfrac{\sqrt{2}}{\sqrt{8} + \sqrt{6}}$ **84.** $\dfrac{6}{\sqrt{2} + \sqrt{3}}$ **85.** $\dfrac{2}{\sqrt{6} - \sqrt{5}}$

86. Geometry One rectangle is $4\sqrt{6}$ feet long and $\sqrt{2}$ feet wide. Another rectangle is $8\sqrt{2}$ feet long and $2\sqrt{6}$ feet wide. How much more area does the larger rectangle cover than the smaller rectangle? (*Hint:* The formula for the area of a rectangle is $A = \ell w$.)

SPIRAL REVIEW

Describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$. *(Lesson 5-9)*

87. $f(x) = -2x + 3; g(x) = -2x - 1$ **88.** $f(x) = 4x; g(x) = 5x$

Factor each polynomial completely. Check your answer. *(Lesson 8-6)*

89. $x^2 + 7x - 30$ **90.** $6x^2 + 11x + 3$ **91.** $x^2 - 16$

92. $3x^2 + 30x + 75$ **93.** $2x^4 - 18$ **94.** $8x^3 - 20x^2 - 12x$

Simplify. All variables represent nonnegative numbers. *(Lesson 11-6)*

95. $\sqrt{360}$ **96.** $\sqrt{\dfrac{72}{16}}$ **97.** $\sqrt{\dfrac{49x^2}{64y^4}}$ **98.** $\sqrt{\dfrac{50a^7}{9a^3}}$

Solving Radical Equations

Objective
Solve radical equations.

Vocabulary
radical equation
extraneous solution

Who uses this?
Meteorologists can use radical equations to estimate the size of a storm. (See Exercise 76.)

A **radical equation** is an equation that contains a variable within a radical. In this course, you will only study radical equations that contain square roots.

Recall that you use inverse operations to solve equations. For nonnegative numbers, squaring and taking the square root are inverse operations. When an equation contains a variable within a square root, square both sides of the equation to solve.

Know it! Note

Power Property of Equality

WORDS	NUMBERS	ALGEBRA
You can square both sides of an equation, and the resulting equation is still true.	$3 = 1 + 2$ $(3)^2 + (1 + 2)^2$ $9 = 9$	If a and b are real numbers and $a = b$, then $a^2 = b^2$.

E X A M P L E 1 **Solving Simple Radical Equations**

Solve each equation. Check your answer.

A $\sqrt{x} = 8$

$(\sqrt{x})^2 = (8)^2$ *Square both sides.*

$x = 64$

Check $\sqrt{x} = 8$

$\sqrt{64}$ | 8 *Substitute 64 for x in the original equation.*

8 | 8 ✓ *Simplify.*

B $6 = \sqrt{4x}$

$(6)^2 = (\sqrt{4x})^2$ *Square both sides.*

$36 = 4x$

$9 = x$ *Divide both sides by 4.*

Check $6 = \sqrt{4x}$

6 | $\sqrt{4(9)}$ *Substitute 9 for x in the original equation.*

6 | $\sqrt{36}$ *Simplify.*

6 | 6 ✓

CHECK IT OUT! Solve each equation. Check your answer.

1a. $\sqrt{x} = 6$ **1b.** $9 = \sqrt{27x}$ **1c.** $\sqrt{3x} = 1$

Some square-root equations do not have the square root isolated. To solve these equations, you may have to isolate the square root before squaring both sides. You can do this by using one or more inverse operations.

EXAMPLE 2 **Solving Radical Equations by Adding or Subtracting**

Solve each equation. Check your answer.

A $\sqrt{x} + 3 = 10$

$\sqrt{x} = 7$ *Subtract 3 from both sides.*

$\left(\sqrt{x}\right)^2 = (7)^2$ *Square both sides.*

$x = 49$

Check $\sqrt{x} + 3 = 10$

$\sqrt{49} + 3 \mid 10$

$7 + 3 \mid 10$

$10 \mid 10$ ✓

B $\sqrt{x - 5} = 4$

$\left(\sqrt{x - 5}\right)^2 = (4)^2$ *Square both sides.*

$x - 5 = 16$

$x = 21$ *Add 5 to both sides.*

Check $\sqrt{x - 5} = 4$

$\sqrt{21 - 5} \mid 4$

$\sqrt{16} \mid 4$

$4 \mid 4$ ✓

C $\sqrt{2x - 1} + 4 = 7$

$\sqrt{2x - 1} = 3$ *Subtract 4 from both sides.*

$\left(\sqrt{2x - 1}\right)^2 = (3)^2$ *Square both sides.*

$2x - 1 = 9$

$2x = 10$ *Add 1 to both sides.*

$x = 5$ *Divide both sides by 2.*

Check $\sqrt{2x - 1} + 4 = 7$

$\sqrt{2(5) - 1} + 4 \mid 7$

$\sqrt{10 - 1} + 4 \mid 7$

$\sqrt{9} + 4 \mid 7$

$3 + 4 \mid 7$

$7 \mid 7$ ✓

CHECK IT OUT! Solve each equation. Check your answer.

2a. $\sqrt{x} - 2 = 1$ **2b.** $\sqrt{x + 7} = 5$ **2c.** $\sqrt{3x + 7} - 1 = 3$

EXAMPLE 3 **Solving Radical Equations by Multiplying or Dividing**

Solve each equation. Check your answer.

A $3\sqrt{x} = 21$

Method 1

$3\sqrt{x} = 21$

$\sqrt{x} = 7$ *Divide both sides by 3.*

$\left(\sqrt{x}\right)^2 = (7)^2$ *Square both sides.*

$x = 49$

Method 2

$3\sqrt{x} = 21$

$\left(3\sqrt{x}\right)^2 = 21^2$ *Square both sides.*

$9x = 441$

$x = 49$ *Divide both sides by 9.*

Check $3\sqrt{x} = 21$

$3\sqrt{49} \mid 21$ *Substitute 49 for x in the original equation.*

$3(7) \mid 21$ *Simplify.*

$21 \mid 21$ ✓

Solve each equation. Check your answer.

B $\dfrac{\sqrt{x}}{3} = 5$

Method 1	Method 2
$\sqrt{x} = 15$ *Multiply both sides by 3.*	$\left(\dfrac{\sqrt{x}}{3}\right)^2 = (5)^2$ *Square both sides.*
$\left(\sqrt{x}\right)^2 = (15)^2$ *Square both sides.*	$\dfrac{x}{9} = 25$
$x = 225$	$x = 225$ *Multiply both sides by 9.*

$\quad$ **Check** $\quad \dfrac{\sqrt{x}}{3} = 5$

$\qquad \dfrac{\sqrt{225}}{3} \;\bigg|\; 5 \qquad$ *Substitute 225 for x in the original equation.*

$\qquad \dfrac{15}{3} \;\bigg|\; 5 \qquad$ *Simplify.*

$\qquad 5 \;\bigg|\; 5 \;✓$

CHECK IT OUT! Solve each equation. Check your answer.

3a. $2\sqrt{x} = 22$ $\qquad$ **3b.** $2 = \dfrac{\sqrt{x}}{4}$ $\qquad$ **3c.** $\dfrac{2\sqrt{x}}{5} = 4$

E X A M P L E 4 **Solving Radical Equations with Square Roots on Both Sides**

Solve each equation. Check your answer.

A $\sqrt{x+1} = \sqrt{3}$

$\quad \left(\sqrt{x+1}\right)^2 = \left(\sqrt{3}\right)^2$ *Square both sides.* $\qquad$ **Check** $\quad \sqrt{x+1} = \sqrt{3}$

$\qquad x + 1 = 3$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \sqrt{2+1} \;\big|\; \sqrt{3}$

$\qquad\qquad x = 2$ *Subtract 1 from both sides.* $\qquad\qquad\qquad\qquad \sqrt{3} \;\big|\; \sqrt{3} \;✓$

B $\sqrt{x+8} - \sqrt{3x} = 0$

$\quad \sqrt{x+8} = \sqrt{3x}$ *Add $\sqrt{3x}$ to both sides.* $\quad$ **Check** $\;\sqrt{x+8} - \sqrt{3x} = 0$

$\quad \left(\sqrt{x+8}\right)^2 = \left(\sqrt{3x}\right)^2$ *Square both sides.* $\qquad\qquad \sqrt{4+8} - \sqrt{3(4)} \;\big|\; 0$

$\qquad x + 8 = 3x$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \sqrt{12} - \sqrt{12} \;\big|\; 0$

$\qquad\qquad 8 = 2x$ *Subtract x from both sides.* $\qquad\qquad\qquad\qquad\qquad 0 \;\big|\; 0 \;✓$

$\qquad\qquad 4 = x$ *Divide both sides by 2.*

CHECK IT OUT! Solve each equation. Check your answer.

4a. $\sqrt{3x+2} = \sqrt{x+6}$ $\qquad\qquad$ **4b.** $\sqrt{2x-5} - \sqrt{6} = 0$

Squaring both sides of an equation may result in an **extraneous solution**—
a number that is not a solution of the original equation.

Suppose your original equation is $x = 3$. $\hfill x = 3$

Square both sides. Now you have a new equation. $\hfill x^2 = 9$

Solve this new equation for x by taking the square root of $\hfill \sqrt{x^2} = \sqrt{9}$
both sides. $\hfill x = 3 \text{ or } x = -3$

Now there are two solutions. One $(x = 3)$ is the original equation. The other
$(x = -3)$ is extraneous—it is not a solution of the original equation. Because of
extraneous solutions, it is important to check your answers.

EXAMPLE **5** **Extraneous Solutions**

Solve $\sqrt{6-x} = x$. Check your answer.

$$\left(\sqrt{6-x}\right)^2 = (x)^2 \qquad \textit{Square both sides.}$$
$$6 - x = x^2$$
$$x^2 + x - 6 = 0 \qquad \textit{Write in standard form.}$$
$$(x-2)(x+3) = 0 \qquad \textit{Factor.}$$
$$x - 2 = 0 \text{ or } x + 3 = 0 \qquad \textit{Zero-Product Property}$$
$$x = 2 \text{ or } \qquad x = -3 \qquad \textit{Solve for x.}$$

Helpful Hint

The equation in Example 5 has one solution, but for some equations, all of the solutions are extraneous. In this case, the equation has no solutions.

Check

$$\dfrac{\sqrt{6-x} = x}{}$$

$$\begin{array}{c|c} \sqrt{6-\mathbf{2}} & \mathbf{2} \\ \sqrt{4} & 2 \\ 2 & 2 \checkmark \end{array} \qquad \textit{Substitute 2 for x in the equation.}$$

$$\dfrac{\sqrt{6-x} = x}{}$$

$$\begin{array}{c|c} \sqrt{6-(\mathbf{-3})} & \mathbf{-3} \\ \sqrt{9} & -3 \\ 3 & -3 \; \textbf{✗} \end{array} \qquad \textit{Substitute −3 for x in the equation.}$$

-3 does not check; it is extraneous. The only solution is 2.

CHECK IT OUT! Solve each equation. Check your answer.

5a. $11 + \sqrt{5x} = 6$ **5b.** $x = \sqrt{-3x - 2}$ **5c.** $x - 2 = \sqrt{x}$

EXAMPLE **6** *Geometry Application*

Geometry

A rectangle has an area of 52 square feet. Its length is 13 feet, and its width is $\sqrt{x}$ feet. What is the value of x? What is the width of the rectangle?

$A = 52 \text{ ft}^2$ | $\sqrt{x}$ ft

13 ft
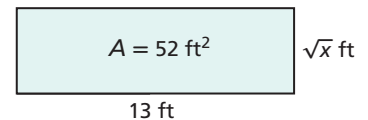
$$A = \ell w \qquad \textit{Use the formula for area of a rectangle.}$$
$$52 = 13\sqrt{x} \qquad \textit{Substitute 52 for A, 13 for ℓ, and } \sqrt{x} \textit{ for w.}$$
$$\dfrac{52}{13} = \dfrac{13\sqrt{x}}{13} \qquad \textit{Divide both sides by 13.}$$
$$4 = \sqrt{x}$$
$$4^2 = \left(\sqrt{x}\right)^2 \qquad \textit{Square both sides.}$$
$$16 = x$$

Check $\quad A = \ell w$

$$52 = 13\sqrt{x}$$

$$\begin{array}{c|c} 52 & 13\sqrt{\mathbf{16}} \\ 52 & 13(4) \\ 52 & 52 \checkmark \end{array} \qquad \textit{Substitute 16 for x in the equation.}$$

The value of x is **16**. The width of the rectangle is $\sqrt{16} = 4$ feet.

CHECK IT OUT! **6.** A rectangle has an area of 15 cm². Its width is 5 cm, and its length is $\left(\sqrt{x+1}\right)$ cm. What is the value of x? What is the length of the rectangle?

THINK AND DISCUSS

1. Compare the two methods used in Example 3A. Which method do you prefer? Why?

2. What is the first step to solve $\sqrt{x-2} + 3 = 8$? Why?

3. GET ORGANIZED Copy and complete the graphic organizer. Write and solve a radical equation, using the boxes to show each step.

Solving Radical Equations

| 1. | → | 2. | → | 3. | → | 4. |
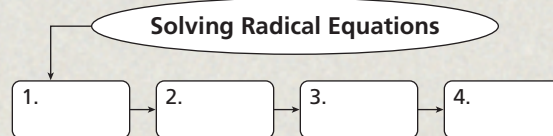
11-9 Exercises
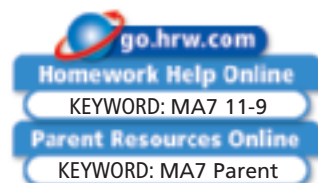
GUIDED PRACTICE

1. Vocabulary Is $x = \sqrt{3}$ a *radical equation*? Why or why not?

SEE EXAMPLE 1
p. 822

Solve each equation. Check your answer.

2. $\sqrt{x} = 7$

3. $4 = \sqrt{-2y}$

4. $\sqrt{20a} = 10$

5. $12 = \sqrt{-x}$

SEE EXAMPLE 2
p. 823

6. $\sqrt{x} + 6 = 11$

7. $\sqrt{2x-5} = 7$

8. $\sqrt{2-a} = 3$

9. $\sqrt{2x} - 3 = 7$

10. $\sqrt{x-2} = 3$

11. $\sqrt{x+3} = 1$

12. $\sqrt{x-1} = 2$

13. $\sqrt{4y+13} - 1 = 6$

SEE EXAMPLE 3
p. 823

14. $-2\sqrt{x} = -10$

15. $\dfrac{\sqrt{a}}{2} = 4$

16. $5\sqrt{-x} = 20$

17. $\dfrac{3\sqrt{x}}{4} = 3$

18. $\dfrac{5\sqrt{x}}{6} = 10$

19. $2\sqrt{x} = 8$

20. $\dfrac{\sqrt{x}}{3} = 3$

21. $\dfrac{3\sqrt{x}}{2} = 1$

22. $13\sqrt{2x} = 26$

23. $\dfrac{\sqrt{x}}{5} = 2$

24. $\dfrac{\sqrt{x-7}}{3} = 1$

25. $4\sqrt{2x-1} = 12$

SEE EXAMPLE 4
p. 824

Solve each equation. Check your answer.

26. $\sqrt{5-x} = \sqrt{6x-2}$

27. $\sqrt{x+7} = \sqrt{3x-19}$

28. $0 = \sqrt{2x} - \sqrt{x+3}$

29. $\sqrt{x-5} = \sqrt{7-x}$

30. $\sqrt{-x} = \sqrt{2x+1}$

31. $\sqrt{3x+1} - \sqrt{2x+3} = 0$

SEE EXAMPLE 5
p. 825

Solve each equation. Check your answer.

32. $\sqrt{x-5} + 5 = 0$

33. $\sqrt{3x} + 5 = 3$

34. $\sqrt{2-7x} = 2x$

35. $x = \sqrt{12+x}$

36. $6 + \sqrt{x-1} = 4$

37. $\sqrt{6-3x} + 2 = x$

38. $\sqrt{x-2} = 2 - x$

39. $10 + \sqrt{x} = 5$

SEE EXAMPLE 6
p. 825

40. Geometry A trapezoid has an area of 14 cm². The length of one base is 4 cm and the length of the other base is 10 cm. The height is $\left(\sqrt{2x+3}\right)$ cm. What is the value of x? What is the height of the trapezoid? (*Hint:* The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$.)

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

41. $\sqrt{3x} = 12$ **42.** $2 = \sqrt{-2x}$ **43.** $\sqrt{-a} = 5$ **44.** $11 = \sqrt{c}$

45. $\sqrt{x-7} = 8$ **46.** $\sqrt{x} - 4 = 0$ **47.** $\sqrt{1-3x} = 5$ **48.** $\sqrt{5x+1} + 2 = 6$

49. $5\sqrt{x} = 30$ **50.** $\dfrac{\sqrt{2x}}{2} = 4$ **51.** $5\sqrt{-x} = 20$ **52.** $3\sqrt{3p} = 9$

Solve each equation. Check your answer.

53. $\sqrt{3x-13} = \sqrt{x+3}$ **54.** $\sqrt{x} - \sqrt{6-x} = 0$ **55.** $\sqrt{x+5} = \sqrt{2x-4}$

56. $\sqrt{4x-2} = \sqrt{3x+4}$ **57.** $\sqrt{5x-6} = \sqrt{16-6x}$ **58.** $\sqrt{12x-3} = \sqrt{4x+93}$

Solve each equation. Check your answer.

59. $\sqrt{x+6} = 1$ **60.** $-2\sqrt{x} = 6$ **61.** $x = \sqrt{2x+15}$ **62.** $\sqrt{6x} + 9 = 2$

63. $\sqrt{4-3x} = x$ **64.** $\sqrt{5x+4} = x - 4$ **65.** $\sqrt{2x+2} = 2x$ **66.** $\sqrt{x+3} + 10 = 7$

67. Geometry A triangle has an area of 60 in². Its base is 10 inches and its height is $\sqrt{x}$ inches. What is the value of x? What is the height of the triangle? (*Hint:* The formula for the area of a triangle is $A = \frac{1}{2}bh$.)

Translate each sentence into an equation. Then solve the equation and check your answer.

68. The square root of three times a number is nine.

69. The difference of the square root of a number and three is four.

70. The square root of the difference of a number and three is four.

71. A number is equal to the square root of the sum of that number and six.

Geometry Find the dimensions of each rectangle given its perimeter.

72.

5 m

$(\sqrt{x+7})$ m

$P = 18$ m

73.

$(\sqrt{x+3})$ in.

1 in.

$P = 8$ in.

74.

$3\sqrt{x}$ cm

$2\sqrt{x}$ cm

$P = 30$ cm
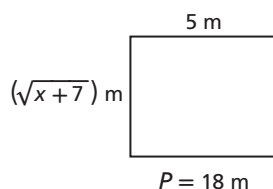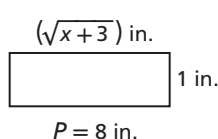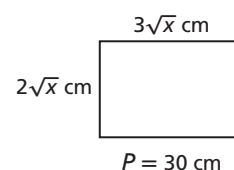
Meteorology
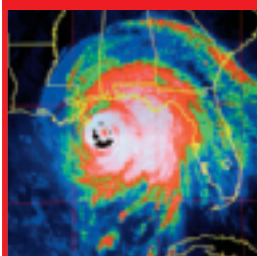
This color-enhanced satellite image is from the National Hurricane Center. Images such as this one show the size of a storm and the area that it covers.

75. Physics The formula $v = \dfrac{\sqrt{2Em}}{m}$ describes the relationship between an object's mass m in kilograms, its velocity v in meters per second, and its kinetic energy E in joules.

  a. A baseball with a mass of 0.14 kg is thrown with a velocity of 28 m/s. How much kinetic energy does the baseball have?

  b. What is the kinetic energy of an object at rest ($v = 0$)?

76. Meteorology The formula $t = \sqrt{\dfrac{d^2}{216}}$ gives the time t in hours that a storm with diameter d miles will last. What is the diameter of a storm that lasts 1 hour? Round your answer to the nearest hundredth.

77. Transportation A sharp curve may require a driver to slow down to avoid going off the road. The equation $v = \sqrt{2.5r}$ describes the relationship between the radius r in feet of an unbanked curve and the maximum velocity v in miles per hour that a car can safely go around the curve. An engineer is designing a highway with a maximum speed limit of 65 mi/h. What is the radius of an unbanked curve for which this is the maximum safe speed?

78. **Write About It** Explain why it is important to check solutions when solving radical equations.

79. **Multi-Step** Solve for x and y in the equations $\sqrt{x} + \sqrt{y} = \sqrt{81}$ and $6\sqrt{y} = 24$. (*Hint:* Solve for y first, and then use substitution to solve for x.)

Tell whether the following statements are *always*, *sometimes*, or *never* true. If the answer is *sometimes*, give one example that is true and one that is false.

80. If $a = b$, then $a^2 = b^2$.

81. If $a^2 = b^2$, then $a = b$.

82. When solving radical equations, the value of the variable is nonnegative.

83. ///**ERROR ANALYSIS**/// Two students solved $\sqrt{5 - x} = \sqrt{x + 9}$. Which is incorrect? Explain the error.

A
$$\sqrt{5 - x} = \sqrt{x + 9}$$
$$\left(\sqrt{5 - x}\right)^2 = \left(\sqrt{x + 9}\right)^2$$
$$5 - x = x + 9$$
$$-2x = 4$$
$$x = -2$$

Check $\sqrt{5 - (-2)}$? $\sqrt{(-2) + 9}$
$\sqrt{7} = \sqrt{7}$ ✔

B
$$\sqrt{5 - x} = \sqrt{x + 9}$$
$$\left(\sqrt{5 - x}\right)^2 = \left(\sqrt{x + 9}\right)^2$$
$$5 - x = x + 9$$
$$4 = 2x$$
$$2 = x$$

Check $\sqrt{5 - 2}$? $\sqrt{2 + 9}$
$\sqrt{3} \neq \sqrt{11}$ ✗
no solution

84. **Estimation** The relationship between a circle's radius and its area can be modeled by $r = \frac{\sqrt{A}}{\pi}$, where r is the radius and A is the area. Solutions to this equation are graphed at right. Use the graph to estimate the radius of a circle with an area of 29 m².

Area and Radius of a Circle
Radius (m) vs Area (m²)

85. **Critical Thinking** Suppose that the equation $\sqrt{-x} = k$ has a solution. What does that tell you about the value of x? the value of k? Explain.
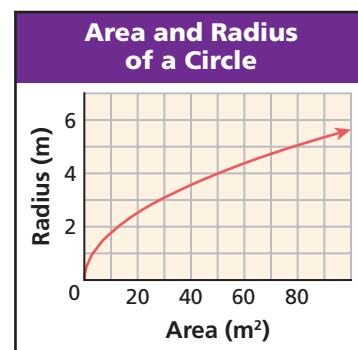
MULTI-STEP TEST PREP

86. This problem will prepare you for the Multi-Step Test Prep on page 830.
   a. The Demon Drop is a free-fall ride at Ohio's Cedar Point amusement park. The maximum speed on the Demon Drop is 55 miles per hour. Convert this speed to feet per second. (*Hint:* There are 5280 feet in a mile.)
   b. The Demon Drop is 131 feet tall. Most of the drop is a vertical free fall, but near the bottom, the track curves so that the cars slow down gradually. The velocity of any object in free fall is $v = \sqrt{8d}$, where v is the velocity in feet per second and d is the distance the object has fallen in feet. Use this equation and your answer to part **a** to estimate the free-fall distance.
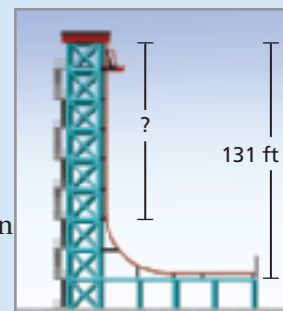
87. Which of the following is the solution of $\sqrt{8-2x}-2=2$?

    (A) −4   (B) −2   (C) 2   (D) 4

88. For which of the following values of k does the equation $\sqrt{x+1}+k=0$ have no real solution?

    (F) −2   (G) −1   (H) 0   (J) 1

89. Which of the following is the solution of $x=\sqrt{12-x}$?

    (A) −4   (B) −3   (C) 3   (D) 4

90. Which of the following is the solution of $\sqrt{x+13}=5\sqrt{x-11}$?

    (F) 9   (G) 12   (H) 16   (J) 17

91. Which of the following is an extraneous solution of $\sqrt{3x-2}=x-2$?

    (A) 1   (B) 2   (C) 3   (D) 6

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

92. $\sqrt{x+3}=x+1$ **93.** $\sqrt{x-1}=x-1$ **94.** $x-1=\sqrt{2x+6}$

95. $\sqrt{x^2+5x+11}=x+3$ **96.** $\sqrt{x^2+9x+14}=x+4$ **97.** $x+2=\sqrt{x^2+5x+4}$

98. Graphing Calculator Solve $\sqrt{2x-2}=-\sqrt{x}$ and check your answer. Then use your graphing calculator for the following:

    a. Graph $y=\sqrt{2x-2}$ and $y=-\sqrt{x}$ on the same screen. Make a sketch of the graphs.

    b. Use the graphs in part **a** to explain your solution to $\sqrt{2x-2}=-\sqrt{x}$.

99. Graphing Calculator Solve $x=\sqrt{x+6}$ and check your answer. Then use your graphing calculator for the following:

    a. Graph $y=x$ and $y=\sqrt{x+6}$ on the same screen. Make a sketch of the graphs.

    b. Use the graphs in part **a** to explain your solution to $x=\sqrt{x+6}$.

100. Find the domain for the function $y=\frac{4}{\sqrt{x-2}}$. Is the domain for this function different from the domain for the function $y=\sqrt{x-2}$? Why or why not?

SPIRAL REVIEW

101. On a map, the distance between two towns is 3.2 inches. If the map uses the scale 2.5 in : 40 mi, what is the actual distance between the towns? *(Lesson 2-6)*

102. In model railroading, O-scale trains use the scale 1 : 48. An O-scale boxcar measures 12.5 inches. How many feet long is the boxcar that it models? *(Lesson 2-6)*

103. The personal identification number (PIN) for a debit card is made up of four numbers. How many PINs are possible? *(Lesson 10-8)*

104. A dessert menu offers 6 different selections. The restaurant offers a dessert sampler that includes small portions of any 4 different choices from the dessert menu. How many different dessert samplers are possible? *(Lesson 10-8)*

Graph each square-root function. *(Lesson 11-5)*

105. $f(x)=\sqrt{x+3}$ **106.** $f(x)=\sqrt{3x-6}$ **107.** $f(x)=2\sqrt{x}+1$

MULTI-STEP TEST PREP

Radical Functions and Equations

Eye in the Sky The London Eye is a giant observation wheel in London, England. It carries people in enclosed capsules around its circumference. Opened on December 31, 1999, to welcome the new millennium, its diameter is 135 meters. On the London Eye, riders can see a distance of 40 kilometers.

1. What is the circumference of the London Eye? Use 3.14 for π.

2. The London Eye's velocity in meters per second can be found using the equation $v = \sqrt{0.001r}$, where r is the radius of the wheel in meters. Find the velocity in meters per second. Round to the nearest hundredth.

3. Another way to find the velocity is to divide the distance around the wheel by the time for the ride. A ride on the London Eye lasts 30 minutes. Use this method to find the velocity of the wheel in meters per second to the nearest hundredth.

4. Are your answers to problems 2 and 3 the same? If not, explain any differences.

5. When a rider is at the highest point on the London Eye, how far is he from the bottom of the ride? Explain.

6. When a rider is at half the maximum height, her distance from the bottom of the ride can be found using the equation $d = \sqrt{2r^2}$. Explain where this equation comes from. Then find this distance. Round to the nearest hundredth.

READY TO GO ON?

Quiz for Lessons 11-5 Through 11-9

✔ 11-5 Square-Root Functions

1. The distance in kilometers that a person can see to the horizon can be approximated by the formula $D = 113\sqrt{h}$, where h is the person's height in kilometers above sea level. What is the distance to the horizon observed by a mountain climber who is 0.3 km above sea level?

Find the domain of each square-root function.

2. $y = \sqrt{3x} - 7$ 3. $y = \sqrt{x - 5}$ 4. $y = \sqrt{2x - 6}$

Graph each square-root function.

5. $f(x) = \sqrt{x - 6}$ 6. $f(x) = \sqrt{x} + 5$ 7. $f(x) = \sqrt{8 - 4x}$

✔ 11-6 Radical Expressions

Simplify. All variables represent nonnegative numbers.

8. $\sqrt{75}$ 9. $\sqrt{\dfrac{300}{3}}$ 10. $\sqrt{a^2 b^3}$ 11. $\sqrt{98xy^2}$

12. $\sqrt{\dfrac{32}{25}}$ 13. $\sqrt{\dfrac{128}{121}}$ 14. $\sqrt{\dfrac{4b^2}{81}}$ 15. $\sqrt{\dfrac{75a^9}{49a^3}}$

16. How long is the diagonal of a rectangular television screen that is 19.2 inches long and 14.4 inches high?

✔ 11-7 Adding and Subtracting Radical Expressions

Simplify each expression.

17. $12\sqrt{7} - 5\sqrt{7}$ 18. $3\sqrt{x} + 3\sqrt{x}$ 19. $\sqrt{12} + \sqrt{75}$

20. $5\sqrt{50} + \sqrt{98}$ 21. $4\sqrt{3} - 3\sqrt{4}$ 22. $\sqrt{98x} + \sqrt{18x} - \sqrt{200x}$

✔ 11-8 Multiplying and Dividing Radical Expressions

Multiply. Write each product in simplest form.

23. $\sqrt{6}\,\sqrt{11}$ 24. $\sqrt{3}\,\sqrt{8}$ 25. $4\sqrt{12x}\,\sqrt{3x}$ 26. $\left(3 - \sqrt{3}\right)\left(5 + \sqrt{3}\right)$

Simplify each quotient.

27. $\dfrac{\sqrt{19}}{\sqrt{3}}$ 28. $\dfrac{\sqrt{14}}{\sqrt{8}}$ 29. $\dfrac{\sqrt{6b}}{\sqrt{8}}$ 30. $\dfrac{\sqrt{27}}{\sqrt{3t}}$

✔ 11-9 Solving Radical Equations

Solve each equation. Check your answer.

31. $\sqrt{x} - 4 = 21$ 32. $-3\sqrt{x} = -12$ 33. $\dfrac{5\sqrt{x}}{2} = 40$

34. $\sqrt{4x - 2} - \sqrt{43 - x} = 0$ 35. $\sqrt{20 + x} = x$ 36. $\sqrt{4x} + 12 = 10$

Rational Exponents

Objective
Simplify expressions containing rational exponents.

Vocabulary
index

You have seen that taking a square root and squaring are inverse operations for nonnegative numbers.

$$\sqrt{49}=7 \qquad \sqrt{225}=15$$
$$7^2=49 \qquad 15^2=225$$

There are inverse operations for other powers as well. For example, $\sqrt[3]{}$ represents a cube root, and it is the inverse of cubing a number. To find $\sqrt[3]{8}$, look for three equal factors whose product is 8. Since $2 \cdot 2 \cdot 2 = 8$, $\sqrt[3]{8} = 2$.

Operation	Inverse	Example
Squaring	Square root $\left(\sqrt{}\right)$	$5^2 = 25 \leftrightarrow \sqrt{25} = 5$
Cubing	Cube root $\left(\sqrt[3]{}\right)$	$2^3 = 8 \leftrightarrow \sqrt[3]{8} = 2$
Raising to the 4th power	Fourth root $\left(\sqrt[4]{}\right)$	$3^4 = 81 \leftrightarrow \sqrt[4]{81} = 3$
Raising to the 5th power	Fifth root $\left(\sqrt[5]{}\right)$	$2^5 = 32 \leftrightarrow \sqrt[5]{32} = 2$
And so on...		

Know it! Note

In the table, notice the small number to the left of the radical sign. It indicates what root you are taking. This is the **index** (plural, *indices*) of the radical. When a radical is written without an index, the index is understood to be 2. Only positive integers greater than or equal to 2 may be indices of radicals.

EXAMPLE 1 **Simplifying Roots**

Simplify each expression.

A $\sqrt{36}$

$\sqrt{36} = 6$ *Think* $\blacksquare^2 = 36$.

B $\sqrt[3]{64}$

$\sqrt[3]{64} = 4$ *Think* $\blacksquare^3 = 64$.

C $\sqrt[4]{1}$

$\sqrt[4]{1} = 1$ *Think* $\blacksquare^4 = 1$.

D $\sqrt[5]{100,000}$

$\sqrt[5]{100,000} = 10$ *Think* $\blacksquare^5 = 100,000$.

CHECK IT OUT! Simplify each expression.

1a. $\sqrt[3]{27}$ 1b. $\sqrt[5]{0}$ 1c. $\sqrt[4]{16}$ 1d. $\sqrt{144}$

You have seen that exponents can be integers. Exponents can also be fractions. What does it mean when an exponent is a fraction? For example, what is the meaning of $3^{\frac{1}{2}}$?

Start by squaring the expression.

Remember!

The Product of Powers Property:
$a^m \cdot a^n = a^{m+n}$

$$\left(3^{\frac{1}{2}}\right)^2 = 3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}$$

$$= 3^{\frac{1}{2}+\frac{1}{2}} \qquad \textit{Product of Powers Property}$$

$$= 3^1$$

$$= 3$$

So, $\left(3^{\frac{1}{2}}\right)^2 = 3$. However, $\left(\sqrt{3}\right)^2 = 3$ also. Since squaring either expression gives a result of 3, it must be true that $3^{\frac{1}{2}} = \sqrt{3}$.

Know it! Note

Definition of $b^{\frac{1}{n}}$

WORDS	NUMBERS	ALGEBRA
A number raised to the power of $\frac{1}{n}$ is equal to the nth root of that number.	$3^{\frac{1}{2}} = \sqrt{3}$ $5^{\frac{1}{4}} = \sqrt[4]{5}$ $2^{\frac{1}{7}} = \sqrt[7]{2}$	If $b \geq 0$ and n is an integer, where $n \geq 2$, then $b^{\frac{1}{n}} = \sqrt[n]{b}$. $b^{\frac{1}{2}} = \sqrt{b}$ $b^{\frac{1}{3}} = \sqrt[3]{b}$ $b^{\frac{1}{4}} = \sqrt[4]{b}$ and so on…

EXAMPLE 2 **Simplifying $b^{\frac{1}{n}}$**

Simplify each expression.

A $125^{\frac{1}{3}}$

$$125^{\frac{1}{3}} = \sqrt[3]{125} \qquad \textit{Use the definition of } b^{\frac{1}{n}}.$$

$$= 5 \qquad \textit{Think } \square^3 = 125.$$

B $64^{\frac{1}{6}}$

$$64^{\frac{1}{6}} = \sqrt[6]{64} \qquad \textit{Use the definition of } b^{\frac{1}{n}}.$$

$$= 2 \qquad \textit{Think } \square^6 = 64.$$

CHECK IT OUT! Simplify each expression.

2a. $121^{\frac{1}{2}}$ **2b.** $81^{\frac{1}{4}}$ **2c.** $256^{\frac{1}{4}}$

You can also have a fractional exponent with a numerator other than 1.

For example, $8^{\frac{2}{3}} = 8^{\frac{1}{3} \cdot 2}$

$$= \left(8^{\frac{1}{3}}\right)^2 \qquad \textit{Power of a Power Property}$$

$$= \left(\sqrt[3]{8}\right)^2 \qquad \textit{Definition of } b^{\frac{1}{n}}$$

$$= (2)^2 \qquad \textit{Think } \square^3 = 8.\ 2^3 = 8,\ so\ \sqrt[3]{8} = 2.$$

$$= 4$$

EXAMPLE 3 **Simplifying Expressions with Rational Exponents**

Simplify each expression.

A $216^{\frac{2}{3}}$

$$216^{\frac{2}{3}} = 216^{\frac{1}{3} \cdot 2}$$

$$= \left(216^{\frac{1}{3}}\right)^2 \qquad \textit{Power of a Power Property}$$

$$= \left(\sqrt[3]{216}\right)^2 \qquad \textit{Definition of } b^{\frac{1}{n}}$$

$$= (6)^2$$

$$= 36$$

B $32^{\frac{4}{5}}$

$$32^{\frac{4}{5}} = 32^{\frac{1}{5} \cdot 4}$$

$$= \left(32^{\frac{1}{5}}\right)^4$$

$$= \left(\sqrt[5]{32}\right)^4$$

$$= (2)^4$$

$$= 16$$

CHECK IT OUT! Simplify each expression.

3a. $16^{\frac{3}{4}}$ **3b.** $1^{\frac{2}{5}}$ **3c.** $27^{\frac{4}{3}}$

EXTENSION

Exercises

Simplify each expression.

1. $\sqrt{169}$ **2.** $\sqrt[6]{1}$ **3.** $\sqrt[4]{625}$ **4.** $\sqrt[3]{1000}$

5. $\sqrt[5]{32}$ **6.** $\sqrt[4]{16}$ **7.** $\sqrt[3]{8000}$ **8.** $\sqrt[3]{0}$

9. $\sqrt[3]{343}$ **10.** $\sqrt[3]{125}$ **11.** $\sqrt[7]{128}$ **12.** $\sqrt[5]{243}$

13. $\sqrt[4]{1296}$ **14.** $\sqrt[6]{729}$ **15.** $\sqrt[3]{512}$ **16.** $\sqrt[3]{1728}$

17. $\sqrt[8]{256}$ **18.** $\sqrt[4]{6561}$ **19.** $\sqrt[6]{4096}$ **20.** $\sqrt[4]{2401}$

21. Give an example which illustrates that a number raised to the power of $\frac{1}{4}$ is the same as the fourth root of that number.

22. **Write About It** You can write $4^{\frac{3}{2}}$ as $4^{3 \cdot \frac{1}{2}}$ or $4^{\frac{1}{2} \cdot 3}$. Use the Power of a Power Property to show that both expressions are equal. Is one method easier than the other? Explain.

23. **Critical Thinking** In this lesson, the Product of Powers Property was used to show that $3^{\frac{1}{2}}$ is the same as $\sqrt{3}$. Use the Power of a Power Property to show this.

24. **Critical Thinking** Compare $n^{\frac{2}{3}}$ and $n^{\frac{3}{2}}$ for values of n greater than 1. When simplifying each of these expressions, will the result be greater than n or less than n? Explain.

25. Mike invested in stock that has been losing value at a rate of 4% each year. The current value of the stock is $10,000. Mike wants to sell it now, but he cannot sell for 9 months. If the stock's value continues to decline at the same rate, how much will it be worth when he sells? Round to the nearest dollar.

Simplify each expression.

26. $8^{\frac{1}{3}}$　　　　**27.** $16^{\frac{1}{2}}$　　　　**28.** $0^{\frac{1}{6}}$　　　　**29.** $81^{\frac{1}{2}}$

30. $216^{\frac{1}{3}}$　　　**31.** $1^{\frac{1}{9}}$　　　　**32.** $625^{\frac{1}{4}}$　　　**33.** $729^{\frac{1}{2}}$

34. $32^{\frac{1}{5}}$　　　　**35.** $196^{\frac{1}{2}}$　　　**36.** $256^{\frac{1}{8}}$　　　**37.** $289^{\frac{1}{2}}$

38. $81^{\frac{3}{4}}$　　　　**39.** $400^{\frac{1}{2}}$　　　**40.** $8^{\frac{5}{3}}$　　　　**41.** $125^{\frac{2}{3}}$

42. $25^{\frac{3}{2}}$　　　　**43.** $36^{\frac{3}{2}}$　　　　**44.** $64^{\frac{4}{3}}$　　　　**45.** $256^{\frac{3}{4}}$

46. $243^{\frac{2}{5}}$　　　**47.** $625^{\frac{3}{4}}$　　　**48.** $4^{\frac{7}{2}}$　　　　**49.** $64^{\frac{5}{6}}$

50. Consumer Economics Hannah is saving to buy a car when she graduates in $2\frac{1}{2}$ years. The car she wants currently costs \$18,500, and the inflation rate is 4%. Use $C = c(1 + r)^y$ to determine the projected cost of the car, where c is the current cost, r is the rate of inflation, and y is the number of years into the future. (*Hint:* Convert the number of years to an improper fraction.)

51. Geometry The formula for the surface area S of a sphere in terms of its volume V is $S = (4\pi)^{\frac{1}{3}} (3V)^{\frac{2}{3}}$. What is the surface area of a sphere that has a volume of 36π cm^3? Use 3.14 for π, and round your answer to the nearest hundredth.

Simplify each expression.

52. $\left(\dfrac{81}{169}\right)^{\frac{1}{2}}$　　**53.** $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}$　　**54.** $\left(\dfrac{256}{81}\right)^{\frac{1}{4}}$　　**55.** $\left(\dfrac{1}{16}\right)^{\frac{1}{2}}$

56. $\left(\dfrac{9}{16}\right)^{\frac{3}{2}}$　　**57.** $\left(\dfrac{8}{27}\right)^{\frac{2}{3}}$　　**58.** $\left(\dfrac{16}{81}\right)^{\frac{3}{4}}$　　**59.** $\left(\dfrac{4}{49}\right)^{\frac{3}{2}}$

60. $\left(\dfrac{4}{25}\right)^{\frac{3}{2}}$　　**61.** $\left(\dfrac{1}{81}\right)^{\frac{3}{4}}$　　**62.** $\left(\dfrac{27}{64}\right)^{\frac{2}{3}}$　　**63.** $\left(\dfrac{8}{125}\right)^{\frac{4}{3}}$

Fill in the boxes to make each statement true.

64. $256^{\frac{\blacksquare}{4}} = 4$　　**65.** $\blacksquare^{\frac{1}{5}} = 1$　　**66.** $225^{\frac{1}{\blacksquare}} = 15$　　**67.** $\blacksquare^{\frac{1}{6}} = 0$

68. $64^{\frac{\blacksquare}{3}} = 16$　　**69.** $\blacksquare^{\frac{3}{4}} = 125$　　**70.** $27^{\frac{4}{\blacksquare}} = 81$　　**71.** $36^{\frac{\blacksquare}{2}} = \dfrac{1}{6}$

72. Finance Laura has \$3000 invested at a rate of 3% compounded yearly. She is excited about the investment and wants to find how much it will be worth in 3 months. Find the value of the investment in 3 months. Round your answer to the nearest dollar.

73. Economics The formula $i = \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}} - 1$ can be used to find the annual rate of inflation i (as a decimal), given the original price p_1 of an item, the new price p_2 of the same item, and the number of years n that have passed. Find the annual rate of inflation if the price of an item increased from \$8.25 to \$14.00 in 5 years. Write your answer as a percent, and round to the nearest whole number.

Challenge Use the properties of exponents to simplify each expression.

74. $\left(a^{\frac{1}{3}}\right)\left(a^{\frac{1}{3}}\right)\left(a^{\frac{1}{3}}\right)$　　　**75.** $\left(x^{\frac{1}{2}}\right)^5\left(x^{\frac{3}{2}}\right)$　　　**76.** $\left(x^{\frac{1}{3}}\right)^4\left(x^5\right)^{\frac{1}{3}}$

Study Guide: Review

Vocabulary

common ratio 766		half-life 783	
compound interest 782		like radicals 811	
exponential decay 783		radical equation 822	
exponential function 772		radical expression 805	
exponential growth 781		radicand 805	
extraneous solution 824		square-root function. 798	
geometric sequence 766			

Complete the sentences below with vocabulary words from the list above.

1. $f(x) = \sqrt{2x}$ is an example of a(n) ___?___ .

2. A(n) ___?___ function has the form $y = a(1 - r)^t$, where $a > 0$.

3. In the formula $a_n = a_1 r^{n-1}$, the variable r represents the ___?___ .

4. $f(x) = 2^x$ is an example of a(n) ___?___ .

11-1 Geometric Sequences (pp. 766–771)

EXAMPLE

■ What is the 10th term of the geometric sequence $-6400, 3200, -1600, 800, \ldots$?

Find the common ratio by dividing consecutive terms.

$\dfrac{3200}{-6400} = -0.5 \quad \dfrac{-1600}{3200} = -0.5$

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_{10} = -6400(-0.5)^{10-1}$ *Substitute.*

$\quad\;\; = -6400(-0.5)^9$ *Simplify.*

$\quad\;\; = 12.5$

EXERCISES

Find the next three terms in each geometric sequence.

5. 1, 3, 9, 27, …

6. 3, −6, 12, −24, …

7. 80, 40, 20, 10, …

8. −1, −4, −16, −64, …

9. The first term of a geometric sequence is 4 and the common ratio is 5. What is the 10th term?

10. What is the 15th term of the geometric sequence 4, 12, 36, 108, …?

11-2 Exponential Functions (pp. 772–778)

EXAMPLE

■ Tell whether the ordered pairs $\{(1, 4), (2, 16), (3, 36), (4, 64)\}$ satisfy an exponential function. Explain.

x	y
1	4
2	16
3	36
4	64

As the x-values increase by a constant amount, the y-values are not multiplied by a constant amount. This function is not exponential.

EXERCISES

Tell whether each set of ordered pairs satisfies an exponential function. Explain.

11. $\{(0, 1), (2, 9), (4, 81), (6, 729)\}$

12. $\{(-2, -8), (-1, -4), (0, 0), (1, 4)\}$

Graph each exponential function.

13. $y = 4^x$

14. $y = \left(\dfrac{1}{4}\right)^x$

11-3 Exponential Growth and Decay (pp. 781–788)

EXAMPLE

- The value of a piece of antique furniture has been increasing at a rate of 2% per year. In 1990, its value was $800. Write an exponential growth function to model the situation. Then find the value of the furniture in the year 2010.

  Step 1 $y = a(1 + r)^t$ *Write the formula.*

  $\quad\quad y = 800(1 + 0.02)^t$ *Substitute.*

  $\quad\quad y = 800(1.02)^t$ *Simplify.*

  Step 2 $y = 800(1.02)^{20}$ *Substitute 20 for t.*

  $\quad\quad \approx 1188.76$ *Simplify and round.*

  The furniture's value will be $1188.76.

EXERCISES

15. The number of students in the book club is increasing at a rate of 15% per year. In 2001, there were 9 students in the book club. Write an exponential growth function to model the situation. Then find the number of students in the book club in the year 2008.

16. The population of a small town is decreasing at a rate of 4% per year. In 1970, the population was 24,500. Write an exponential decay function to model the situation. Then find the population in the year 2020.

11-4 Linear, Quadratic, and Exponential Models (pp. 789–795)

EXAMPLE

- Use the data in the table to describe how Jasmin's debt is changing. Then write a function that models the data. Use your function to predict Jasmin's debt after 8 years.

Jasmin's Debt	
Years	**Debt ($)**
1	130
2	260
3	520
4	1040

$+1$ $\times 2$
$+1$ $\times 2$
$+1$ $\times 2$

Jasmin's debt doubles every year.

For a constant change in time $(+1)$, there is a constant ratio of 2, so the data is exponential.

$\quad y = ab^x$ *Write the general form.*

$\quad y = a(2)^x$ *Substitute 2 for b.*

$130 = a(2)^1$ *Substitute $(1, 130)$ for x and y.*

$\quad a = 65$ *Solve for a.*

$\quad y = 65(2)^x$ *Replace a and b in $y = ab^x$.*

$\quad y = 65(2)^8$ *Substitute 8 for x.*

$\quad y = 16{,}640$ *Simplify with a calculator.*

Jasmin's debt in 8 years will be $16,640.

EXERCISES

Graph each data set. Which kind of model best describes the data?

17. $\{(-2, -12), (-1, -3), (0, 0), (1, -3), (2, -12)\}$

18. $\{(-2, -2), (-1, 2), (0, 6), (1, 10), (2, 14)\}$

19. $\left\{\left(-2, -\frac{1}{4}\right), \left(-1, -\frac{1}{2}\right), (0, -1), (1, -2), (2, -4)\right\}$

Look for a pattern in each data set to determine which kind of model best describes the data.

20. $\{(0, 2), (1, 6), (2, 18), (3, 54), (4, 162)\}$

21. $\{(0, 0), (2, -20), (4, -80), (6, -180), (8, -320)\}$

22. $\{(-8, 5), (-4, 3), (0, 1), (4, -1), (8, -3)\}$

23. Write a function that models the data. Then use your function to predict how long the humidifier will produce steam with 10 quarts of water.

Input and Output of a Humidifier	
Water Volume (qt)	**Steam Time (h)**
3	4.5
4	6
5	7.5
6	9

11-5 Square-Root Functions (pp. 798–803)

EXAMPLE

■ **Graph** $f(x) = 3\sqrt{x-2}$.

Step 1 Find the domain of the function.
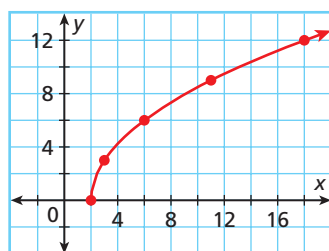
$x - 2 \geq 0$ *The radicand must be greater*

$\quad\ x \geq 2$ *than or equal to 0.*

Step 2 Generate ordered pairs.

x	$f(x) = 3\sqrt{x-2}$
2	0
3	3
6	6
11	9
18	12

Choose x-values greater than or equal to 2 that form a perfect square under the radical sign.

Step 3 Plot and connect the points.

EXERCISES

24. If you know the surface area S of a cube, you can use the formula $\ell = \sqrt{\frac{S}{6}}$ to find the length ℓ of a side. What is the side length of a cube whose surface area is 135 cm²? Round your answer to the nearest hundredth of a centimeter.

Find the domain of each square-root function.

25. $y = \sqrt{x} + 5$ **26.** $y = \sqrt{x+4}$

27. $y = 8 - \sqrt{3x}$ **28.** $y = 2\sqrt{x+2}$

29. $y = 1 + \sqrt{3x-4}$ **30.** $y = \sqrt{2x+6}$

31. $y = \sqrt{2x-7}$ **32.** $y = \sqrt{5x+18}$

33. $y = \sqrt{4x-3}$ **34.** $y = 3\sqrt{x-1}$

Graph each square-root function.

35. $f(x) = \sqrt{x} + 8$ **36.** $f(x) = \sqrt{x-3}$

37. $f(x) = -\sqrt{2x}$ **38.** $f(x) = \sqrt{x} - 1$

39. $f(x) = 2\sqrt{x+3}$ **40.** $f(x) = \sqrt{5-x}$

41. $f(x) = \sqrt{7-4x}$ **42.** $f(x) = 3\sqrt{x-1}$

43. $f(x) = 1 + \sqrt{x+1}$ **44.** $f(x) = \frac{1}{2}\sqrt{x-2}$

11-6 Radical Expressions (pp. 805–810)

EXAMPLES

Simplify. All variables represent nonnegative numbers.

■ $\sqrt{50x^4}$

$\quad \sqrt{(25)(2)x^4}$ *Factor the radicand.*

$\quad \sqrt{25}\,\sqrt{2}\,\sqrt{x^4}$ *Use the Product Property.*

$\quad 5x^2\sqrt{2}$ *Simplify.*

■ $\sqrt{\dfrac{16m^6}{64m^3}}$

$\quad \sqrt{\dfrac{m^3}{4}}$ *Simplify the radicand.*

$\quad \dfrac{\sqrt{m^3}}{\sqrt{4}}$ *Use the Quotient Property.*

$\quad \dfrac{\sqrt{m^2}\,\sqrt{m}}{\sqrt{4}}$ *Use the Product Property.*

$\quad \dfrac{m\sqrt{m}}{2}$ *Simplify.*

EXERCISES

Simplify. All variables represent nonnegative numbers.

45. $\sqrt{121}$ **46.** $\sqrt{n^4}$

47. $\sqrt{(x+3)^2}$ **48.** $\sqrt{\dfrac{75}{3}}$

49. $\sqrt{36d^2}$ **50.** $\sqrt{y^6x}$

51. $\sqrt{12}$ **52.** $\sqrt{32ab^5}$

53. $\sqrt{\dfrac{5}{4}}$ **54.** $\sqrt{\dfrac{t^3}{100t}}$

55. $\sqrt{\dfrac{8}{18}}$ **56.** $\sqrt{\dfrac{32p^4}{49}}$

57. $\sqrt{\dfrac{s^2t^9}{s^4}}$ **58.** $\sqrt{\dfrac{72b^6}{225}}$

11-7 Adding and Subtracting Radical Expressions (pp. 811–815)

EXAMPLE

■ Simplify $\sqrt{50x} - \sqrt{2x} + \sqrt{12x}$.

$$\sqrt{50x} - 1\sqrt{2x} + \sqrt{12x}$$

$$\sqrt{(25)(2)x} - 1\sqrt{2x} + \sqrt{(4)(3)x}$$

$$\sqrt{25}\sqrt{2x} - 1\sqrt{2x} + \sqrt{4}\sqrt{3x}$$

$$5\sqrt{2x} - 1\sqrt{2x} + 2\sqrt{3x}$$

$$4\sqrt{2x} + 2\sqrt{3x}$$

EXERCISES

Simplify each expression.

59. $6\sqrt{7} + 3\sqrt{7}$

60. $4\sqrt{3} - \sqrt{3}$

61. $3\sqrt{2} + 2\sqrt{3}$

62. $9\sqrt{5t} - 8\sqrt{5t}$

63. $\sqrt{50} - \sqrt{18}$

64. $\sqrt{12} + \sqrt{20}$

65. $\sqrt{20x} - \sqrt{80x}$

66. $4\sqrt{54} - \sqrt{24}$

11-8 Multiplying and Dividing Radical Expressions (pp. 816–821)

EXAMPLES

■ Multiply $(\sqrt{3} + 6)^2$. Write the product in simplest form.

$$(\sqrt{3} + 6)^2$$

$(\sqrt{3} + 6)(\sqrt{3} + 6)$ *Expand the expression.*

$3 + 6\sqrt{3} + 6\sqrt{3} + 36$ *Use the FOIL method.*

$39 + 12\sqrt{3}$ *Simplify.*

■ Simplify the quotient $\frac{\sqrt{5}}{\sqrt{3}}$.

$$\frac{\sqrt{5}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{15}}{3}$$ *Rationalize the denominator.*

EXERCISES

Multiply. Write each product in simplest form.

67. $\sqrt{2}\sqrt{7}$

68. $\sqrt{3}\sqrt{6}$

69. $3\sqrt{2x}\sqrt{14}$

70. $(5\sqrt{6})^2$

71. $\sqrt{2}(4 - \sqrt{8})$

72. $(8 + \sqrt{7})^2$

Simplify each quotient.

73. $\frac{4}{\sqrt{5}}$

74. $\frac{a\sqrt{9}}{\sqrt{2}}$

75. $\frac{\sqrt{8}}{2\sqrt{6}}$

76. $\frac{\sqrt{5}}{\sqrt{2n}}$

77. $\frac{\sqrt{18}}{\sqrt{12}}$

78. $\frac{-3}{\sqrt{3}}$

11-9 Solving Radical Equations (pp. 822–829)

EXAMPLE

■ Solve $\sqrt{4x + 1} - 8 = -3$. Check your answer.

$$\sqrt{4x + 1} - 8 = -3$$

$\sqrt{4x + 1} = 5$ *Add 8 to both sides.*

$(\sqrt{4x + 1})^2 = (5)^2$ *Square both sides.*

$4x + 1 = 25$

$4x = 24$ *Subtract 1 from both sides.*

$x = 6$ *Divide both sides by 4.*

Check $\sqrt{4x + 1} - 8 = -3$

$$\sqrt{4(6) + 1} - 8 \;\big|\; -3$$

$$\sqrt{25} - 8 \;\big|\; -3$$

$$5 - 8 \;\big|\; -3$$

$$-3 \;\big|\; -3 \;\checkmark$$

EXERCISES

Solve each equation. Check your answer.

79. $\sqrt{x} = 8$

80. $\sqrt{2x} = 4$

81. $\sqrt{x + 6} = 3$

82. $-3\sqrt{x} = -15$

83. $3\sqrt{-x} = 27$

84. $\frac{4\sqrt{x}}{5} = 8$

85. $\sqrt{x + 1} = \sqrt{3x - 5}$

86. $\sqrt{x - 2} + 4 = 3$

87. $12 = 4\sqrt{2x + 1}$

88. $\sqrt{x - 5} = \sqrt{7 - x}$

89. $\sqrt{x + 2} = 3$

90. $\sqrt{2x - 3} = 4$

91. $4\sqrt{x - 3} = 12$

92. $\sqrt{x + 6} = x$

93. $\sqrt{3x + 4} = x$

94. $\sqrt{2x + 6} = x - 1$

CHAPTER TEST

Find the next three terms in each geometric sequence.

1. 2, 6, 18, 54, …

2. 4800, 2400, 1200, 600, …

3. $-4, 20, -100, 500, …$

4. Communication If school is cancelled, the school secretary calls 2 families. Each of those families calls 2 other families. In the third round of calls, each of the 4 families calls 2 more families. If this pattern continues, how many families are called in the seventh round of calls?

Graph each exponential function.

5. $y = -2(4)^x$

6. $y = 3(2)^x$

7. $y = 4\left(\frac{1}{2}\right)^x$

8. $-\left(\frac{1}{3}\right)^x$

9. A teacher is repeatedly enlarging a diagram on a photocopier. The function $f(x) = 3(1.25)^x$ represents the length of the diagram, in centimeters, after x enlargements. What is the length after 5 enlargements? Round to the nearest centimeter.

10. Chelsea invested $5600 at a rate of 3.6% compounded quarterly. Write a compound interest function to model the situation. Then find the balance after 6 years.

11. The number of trees in a forest is decreasing at a rate of 5% per year. The forest had 24,000 trees 15 years ago. Write an exponential decay function to model the situation. Then find the number of trees now.

Look for a pattern in each data set to determine which kind of model best describes the data.

12. $\{(-10, -17), (-5, -7), (0, 3), (5, 13), (10, 23)\}$

13. $\{(1, 3), (2, 9), (3, 27), (4, 81), (5, 243)\}$

14. Use the data in the table to describe how the bacteria population is changing. Then write a function that models the data. Use your function to predict the bacteria population after 10 hours.

Bacteria Population				
Time (h)	0	1	2	3
Bacteria	6	18	54	162

Find the domain of each square-root function.

15. $y = 6 + \sqrt{x}$

16. $y = -2\sqrt{x + 9}$

17. $y = x + \sqrt{3x - 3}$

Graph each square-root function.

18. $f(x) = \sqrt{x} + 2$

19. $f(x) = \sqrt{x - 1}$

20. $f(x) = -3\sqrt{2x}$

Simplify. All variables represent nonnegative numbers.

21. $\sqrt{27}$

22. $\sqrt{75m^4}$

23. $\sqrt{\dfrac{x^6}{y^2}}$

24. $\sqrt{\dfrac{p^9}{144p}}$

25. $4\sqrt{10} - 2\sqrt{10}$

26. $5\sqrt{3y} + \sqrt{3y}$

27. $\sqrt{8} - \sqrt{50}$

28. $2\sqrt{75} - \sqrt{32} + \sqrt{48}$

29. $\sqrt{2}\sqrt{3m}$

30. $\dfrac{\sqrt{128d}}{\sqrt{5}}$

31. $\sqrt{3}(\sqrt{21} - 2)$

32. $(\sqrt{3} - 2)(\sqrt{3} + 4)$

Solve each equation. Check your answer.

33. $\sqrt{2x} = 6$

34. $\sqrt{3x + 4} - 2 = 5$

35. $\dfrac{2\sqrt{x}}{3} = 8$

36. $\sqrt{5x + 1} = \sqrt{2x - 2}$

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

Colleges use standardized test scores to confirm what your academic record indicates. Because courses and instruction differ from school to school, standardized tests are one way in which colleges try to compare students fairly when making admissions decisions.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

HOT TIP! You will need to use a calculator on the SAT Mathematics Subject Tests. If you do not already have a graphing calculator, consider getting one because it may give you an advantage when solving some problems. Spend time getting used to your calculator before the test.

---

1. What is the domain of $y = \sqrt{x - 4}$?

   (A) $x \geq -2$

   (B) $x \geq 2$

   (C) $x \geq -4$

   (D) $x \geq 4$

   (E) $x > 4$

---

2. $\dfrac{\sqrt{8}\sqrt{3}}{\sqrt{5}} =$

   (A) $2\sqrt{3}$

   (B) $\dfrac{2\sqrt{3}}{5}$

   (C) $\sqrt{12}$

   (D) $\dfrac{4\sqrt{30}}{5}$

   (E) $\dfrac{2\sqrt{30}}{5}$

---

3. If $\dfrac{\sqrt{6 - 3x}}{5} = 3$, what is the value of x?

   (A) -3

   (B) -13

   (C) -73

   (D) -77

   (E) -89

---

4. The third term of a geometric sequence is 32 and the fifth term is 512. What is the eighth term of the sequence?

   (A) 544

   (B) 1232

   (C) 8192

   (D) 32,768

   (E) 2,097,152

---

5. A band releases a new CD and tracks its sales. The table shows the number of copies sold each week (in thousands). Which type of function best models this data?

CD Sales	
Week	**Copies Sold (thousands)**
1	129.5
2	155
3	179.5
4	203
5	225.5
6	247

   (A) Linear function

   (B) Quadratic function

   (C) Exponential function

   (D) Square-root function

   (E) Absolute-value function

Multiple Choice: *None of the Above* or *All of the Above*

In some multiple-choice test items, one of the options is *None of the above* or *All of the above*. To answer these types of items, first determine whether each of the other options is true or false. If you find that more than one option is true, then the correct choice is likely to be *All of the above*. If none of the options are true, the correct choice is *None of the above*.

If you do not know how to solve the problem and have to guess, *All of the above* is most often correct, and *None of the above* is usually incorrect.

EXAMPLE 1

There are 8 players on the chess team. Which of the following expressions gives the number of ways in which the coach can choose 2 players to start the game?

Ⓐ $_8C_2$

Ⓑ $\dfrac{8!}{2!(6!)}$

Ⓒ 28

Ⓓ All of the above

Notice that choice D is All of the above. *This means that you must look at each option. As you consider each option, mark it true or false in your test booklet.*

A Because order does not matter, this is a combination problem. The number of combinations of 8 players, taken 2 at a time, is given by $_nC_r$, where $n = 8$ and $r = 2$, so $_8C_2$ is a correct model of the combination. Choice A is true.

B The number of combinations of 8 players, taken 2 at a time, is given by $_nC_r = \dfrac{n!}{r!(n-r)!}$, where $n = 8$ and $r = 2$.

$$_nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{8!}{2!(8-2)!} = \dfrac{8!}{2!(6)!}$$

Choice B is a correct model of the combination. Choice B is also true. The answer is likely to be choice D, *All of the above*, but you should also check whether choice C is true.

C The number of combinations of 8 players, taken 2 at a time, is given by $_nC_r = \dfrac{n!}{r!(n-r)!}$, where $n = 8$ and $r = 2$.

$$_nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{8!}{2!(8-2)!} = \dfrac{8!}{2!(6)!} = 28$$

Choice C is a correct model of the combination. Choice C is true as well.

Because A, B, and C are all true, the correct response is D, *All of the above*.

Read each test item and answer the questions that follow.

Item A

The mean score on a test is 68. Which CANNOT be true?

(A) Every score is 68.

(B) Half of the scores are 68, and half of the scores are 0.

(C) Half of the scores are 94, and half of the scores are 42.

(D) None of the above

---

1. What is the definition of *mean*?

2. If you find that an option is true, is that the correct response? Explain.

3. Willie determined that A and C could both be true, so he chose D as his response. Do you agree? Why or why not?


Item B

What is the probability of rolling a 2 on a number cube?

(F) $16.\overline{6}\%$

(G) $1 - P(\text{rolling } 1, 3, 4, 5, \text{ or } 6)$

(H) $\dfrac{1}{6}$

(J) All of the above

---

4. What is the complement of rolling a 2? Is choice G correct? Explain.

5. If you roll a number cube, how many possible outcomes are there? How does this information help you solve this problem?

6. Is the value given in choice H equivalent to any other choice? If so, which one(s)?

7. How many choices are true? What is the correct response to the test item?

Item C

Suppose that a dart lands at a random point on the circular dartboard. Find the probability that the dart does NOT land inside the center circle.

3 in. 1 in.

(A) $\dfrac{1}{9}$

(B) 8π
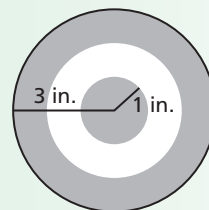
(C) $\dfrac{8}{9}$

(D) None of the above

---

8. Kyle finds that choices A and B are both false. To save time, he selects choice D as his answer because he figures it is likely that choice C will also be false. Do you think Kyle made a wise decision? Why or why not?

9. What is the formula for the area of a circle? What is the total area of this dartboard? How can you determine the area of the dartboard outside the center circle?

10. Determine whether choices A, B, and C are true, and then give the correct response to this test item.

Item D

Each gym member receives a 3-digit locker combination. Digits are from 0–9 and may repeat. What is the probability that you will receive a code consisting of three identical numbers?

(F) $\dfrac{1}{100}$

(G) $\dfrac{10}{{}_{10}C_3}$

(H) ${}_{10}P_3 = \dfrac{10!}{7!}$

(J) All of the above

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11. How can you determine whether choice J is the correct response to the test item?

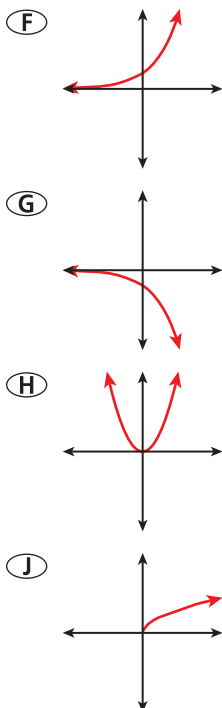12. Are the values given in choices F, G, and H equivalent? What does this tell you about choice J?

STANDARDIZED TEST PREP

CUMULATIVE ASSESSMENT, CHAPTERS 1–11

Multiple Choice

1. A sequence is defined by the rule $a_n = -3(2)^{n-1}$. What is the 5th term of the sequence?

   (A) 5

   (B) −30

   (C) −48

   (D) −216

2. Which could be the graph of $y = -2^x$?

   (F)

   (G)

   (H)

   (J)

3. What is the slope of the line described by $4x - 3y = 12$?

   (A) 4

   (B) 3

   (C) $\frac{4}{3}$

   (D) $-\frac{3}{4}$

4. The odds in favor of a spinner landing on blue are 2:7. What is the probability of the spinner NOT landing on blue?

   (F) $\frac{7}{9}$

   (H) $\frac{2}{7}$

   (G) $\frac{5}{7}$

   (J) $\frac{2}{9}$

5. Which rule can be used to find any term in the sequence 8, 4, 2, 1, …?

   (A) $a_n = 8\left(\frac{1}{2}\right)^n$

   (C) $a_n = \left(\frac{1}{2}\right)^{n-1}$

   (B) $a_n = 2(1)^n$

   (D) $a_n = 8\left(\frac{1}{2}\right)^{n-1}$

6. Jerome walked on a treadmill for 45 minutes at a speed of 4.2 miles per hour. Approximately how far did Jerome walk?

   (F) 1.89 miles

   (G) 2.1 miles

   (H) 3.15 miles

   (J) 5.6 miles

7. What is the solution to the system of equations below?

   $$\begin{cases} 2x - y = 2 \\ y = 3x - 5 \end{cases}$$

   (A) (4, 6)

   (C) (2, 1)

   (B) (3, 4)

   (D) (0, 2)

8. What is the complete factorization of $2x^3 + 18x$?

   (F) $2x(x^2 + 9)$

   (G) $2x(x + 3)^2$

   (H) $2x(x + 3)(x - 3)$

   (J) $2(x^3 + 18)$

9. Which ordered pair lies on the graph of $y = 3(2)^{x+1}$?

   (A) (−1, 0)

   (C) (1, 12)

   (B) (0, 9)

   (D) (3, 24)

10. Which shows the product of 5.1×10^4 and 3×10^9 written in scientific notation?

- F 1.53×10^{12}
- G 1.53×10^{14}
- H 15.3×10^{12}
- J 15.3×10^{13}

11. Which is an arithmetic sequence?

- I   3, 6, 9, 12, 15, …
- II   1, 10, 100, 1000, …
- III   4000, −2000, 1000, −500, …

- A I only
- B II and III
- C I and III
- D III only

Gridded Response

12. The function $f(x) = 30{,}000(0.8)^x$ gives the value of a vehicle, where x is the number of years after purchase. According to the function, what will be the value of the car in dollars 8 years after purchase? Round your answer to the nearest whole dollar.

13. The graph of $f(x)$ is shown below. How many zeros does $f(x)$ have?
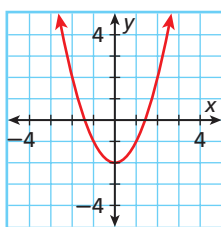
14. Rosalind purchased a sewing machine at a 20%-off sale. The original selling price of the machine was $340. What was the sale price in dollars?

15. Two planes leave an airport, one heading due north and the other heading due west. After several minutes, the first plane is 12 miles north of the airport, and the second plane is 15 miles west of the airport. Estimate the distance between the two planes to the nearest hundredth of a mile.

Short Response

16. The table shows the number of people newly infected by a certain virus with one person as the original source.

Week	1	2	3	4	5	6	7
Newly Infected People	1	3	9	27	81	243	729

- a. Is the data set best described by a linear function, a quadratic function, or an exponential function? Write a function to model the data.
- b. Use the function to predict the number of people that will become infected in the 10th week. Show your work.

17. Ella and Mia went on a camping trip. The total cost for their trip was $124, which the girls divided evenly. Ella paid for 4 nights at the campsite and $30 for supplies. Mia paid for 2 nights at the campsite and $46 for supplies.

- a. Write an equation that could be used to find the cost of one night's stay at the campsite. Explain what each variable in your equation represents.
- b. Solve your equation from part **a** to find the cost of one night's stay at the campsite. Show your work.

Extended Response

18. Regina is beginning a training program to prepare for a race. In week 1, she will run 3 miles during each workout. Each week thereafter, she plans to increase the distance of her runs by 20%.

- a. Write an equation to show the number of miles Regina plans to run during her workouts each week. Use the variable n to represent the week number.
- b. Make a table of values to show the distances of the runs in Regina's workouts for the first 6 weeks. Round each distance to the nearest hundredth of a mile. Then graph the points.
- c. Explain how to use your equation from part **a** to determine how many miles Regina will run during each workout in week 8. Then find this number.
- d. Explain how to the use your graph from part **b** to determine how many miles Regina will run during each workout in week 8. How does the graph verify the answer you found in part **c**?